1. 18.757, Homework 2

Due Thursday, February 21.

1. Suppose (π, V_π) is a finite-dimensional irreducible representation of a group G over a field k. Prove that the dual representation

$$V_\pi^* = \mathrm{Hom}_k(V_\pi, k), \qquad [\pi^*(g)\xi](v) = \xi(\pi(g^{-1})v)$$

(for $g \in G$, $\xi \in V_\pi^*$, and $v \in V_\pi$) is an irreducible representation of G.

2. Suppose k is a field, and D is an arbitrary division algebra over k (meaning that k is contained in the center of D). Find a group G and an irreducible k-representation (π, V_π) of G, with the property that $D_\pi = D$. (Recall that I defined

$$D_\pi = \mathrm{Hom}_{kG}(V_\pi, V_\pi),$$

with the peculiar multiplication given by composition of linear transformations *in the opposite order*:

$$(T \cdot S)(v) = S(T(v)).$$

The purpose of the peculiar definition is to make V_π a *right* vector space over D_π.)