

**18.755 eleventh problems. Due on Gradescope
Wednesday, April 29, 2020, 16:00 Eastern time.**

1. Find all root data living on the dual lattices $X_* = \mathbb{Z}^2$, $X^* = \mathbb{Z}^2$ with the property that R contains the two roots $\alpha = (1, 0)$, $\beta = (0, 1)$.

Hint: this is NOT the same problem you solved last week.

2. Let

$$X_* = \{(x_1, x_2, x_3, x_4) \in \mathbb{Z}^4 \mid x_1 + x_2 + x_3 + x_4 \in 2\mathbb{Z}\},$$

a lattice of rank four. You may assume that the dual lattice is

$$X^* = \{(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \mid (\text{all } \lambda_j \in \mathbb{Z}) \text{ or } (\text{all } \lambda_j \in \mathbb{Z} + 1/2)\}.$$

There is a root datum living on these lattices with

$$R_0^\vee = \{\pm e_i \pm e_j \mid 1 \leq i \neq j \leq 4\} = R_0,$$

the bijection between roots and coroots being the “obvious” one. You may assume that

$$(X_*, R_0^\vee, X^*, R_0)$$

is actually a root datum. (It’s the root datum for the compact group $O(8)$, as we more or less calculated in class.) Find all root data living on X_* and X^* with the property that

$$R_0^\vee \subset R^\vee, \quad R_0 \subset R.$$

Both problems ask you to find **all** of something. Serious partial credit for finding **some**; so if classification seems inaccessible, try invention!