18.745 Problem Set 5 **due in class 3/10/15**

1. Let $\mathfrak{n}$ be the three-dimensional Heisenberg Lie algebra (over any field k), with basis $\{X, Y, Z\}$ and commutation relations

$$[X, Y] = Z, \qquad [X, Z] = 0, \qquad [Y, Z] = 0.$$

Prove that every symmetric invariant bilinear form B on $\mathfrak{n}$ is degenerate.

2. Give an example of a nonabelian nilpotent Lie algebra $\mathfrak{g}$ and a nondegenerate symmetric invariant bilinear form B on $\mathfrak{g}$.

3. Let $\mathfrak{n}$ be the three-dimensional Heisenberg Lie algebra (still over any field k) Calculate the Lie algebra cohomology

$$H^p(\mathfrak{n}, k)$$

for $0 \le p \le 3$.