18.745 Problem Set 3 **due in class 2/24/15**

1. Prove that the Lie algebra $\mathfrak{g}$ is associative if and only if the third term $\mathfrak{g}^2 =_{\text{def}} [\mathfrak{g}, [\mathfrak{g}, \mathfrak{g}]]$ in the lower central series is equal to zero.

2. Two of the most basic properties of algebras are *commutativity* and *associativity*. There is a very easy to state and understand example of an algebra that is associative but not commutative: the algebra $\mathrm{End}(V)$ of linear transformations on a vector space V of dimension at least two. (Associativity is true because composition of functions is always associative; but commutativity is false because the matrices $S = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $T = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ don't commute with each other: precisely, $ST = T$, but $TS = 0$.) Give the simplest example you can of an algebra that is commutative but not associative.

3. The proof of Lie's theorem on pages 94–95 of the text uses the fact that $\mathbb{C}$ has characteristic zero. Give an example of a solvable Lie algebra $\mathfrak{g}$ over an algebraically closed k of characteristic 2, and an irreducible 2-dimensional representation of $\mathfrak{g}$.