18.700 Problem Set 1
Due in class Monday, September 16

1. (4 points) Consider the set of complex numbers

$$G = \{a + bi \mid a, b \in \mathbb{Q}\}.$$

(The G stands for *Gauss*; these numbers might be called *Gaussian rational numbers*, although I don't know if they actually are.) Is G a field (with the same addition and multiplication operations as in $\mathbb{C}$)? For a question like this, you should either explain why all the axioms for a field are satisfied (you can assume that they hold for $\mathbb{C}$), or else explain why one of the axioms fails. A few sentences could be enough to write.

2. (4 points) Consider the set of complex numbers

$$M = \{r_0 + r_1 e^{i\pi/2} \mid r_0, r_1 \in \mathbb{Q}\}.$$

Is M a field?

3. (4 points) Consider the set of complex numbers

$$P = \{re^{2\pi i\theta} \mid r,\ \theta \in \mathbb{Q}\}.$$

Is P a field?

4. (4 points) The vector space $V = (\mathbb{F}_2)^2$ has exactly four vectors $(0,0)$, $(0,1)$, $(1,0)$, and $(1,1)$; so V has exactly $2^4 = 16$ subsets. How many of these 16 subsets are linearly independent? How many bases does V have? For a question like this, you might write some words explaining why some kinds of subset cannot possibly be linearly independent ("the vector $(1,1)$ is in the pay of Big Oil, and so cannot be part of any linearly independent set"). After this you might be left with just a few cases; you could perhaps say a few words about why each of these is or is not linearly independent.

5. (4 points) The set

$$W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y + z + w = 0\}$$

is a subspace of $\mathbb{R}^4$. Find a basis of W.