18.440 Problem Set 5

Due in class Monday October 6; late work will not be accepted. You can discuss problems with anyone, but you should write solutions entirely on your own.

1. (30 points) The setting is that of problem 77 on page 180 (concerning a way to inspect lots of transistors).

a) What proportion of lots is accepted?
b) What is the expected number of good transistors in an accepted lot?
c) What is the expected number of good transistors acquired by the purchaser per lot inspected?
d) Suppose that a lot of transistors costs $10, and that the purchaser pays only for accepted lots. Suppose also that inspection process costs $.50 per lot (a cost that is incurred even if the lot is rejected). What is the expected cost to the purchaser per lot inspected?
e) What is the expected cost per good transistor to the purchaser?
f) Suppose the purchaser simply took all lots: no inspection at all. With this system, what would the expected cost per good transistor be? Any comments on the inspection method?

2. (20 points) Problem 20 on page 181 of the text. Explain what is wrong with the reasonings that you reject.

3. (30 points)

a) Suppose that Y is a discrete random variable with expectation 0 and variance 1. Show that for any positive number t,

$$P(|Y| \geq 10) \leq 1/100.$$

(Hint: in the notation I've been using in class, list the values of Y as $a_1, a_2, \ldots$, with corresponding probabilities $p_1, p_2, \ldots$. The probability in the problem is

$$\sum_{|a_i| \geq 10} p_i.$$

Each value value a_i in this sum must contribute at least 100 times its probability to the variance of Y.)

b) Suppose that a fair coin is tossed n times. Let X be the number of heads. Show that

$$P(|X - n/2| \geq 5\sqrt{n}) \leq 1/100.$$

For $10,000$ tosses, this says that the odds are at least 99 to 1 that the number of heads is between 4500 and 5500. (Hint: consider the random variable $Y = \frac{X-n/2}{\sqrt{n}/2}$.)

4. (20 points) Problem 13 on page 181 of the text. (I found the text a little dense on this one. Another way to state the problem is this: suppose X is a binomial random variable with a known parameter n and an unknown parameter p. Under these circumstances, the probability that X is equal to k becomes a function of the unknown parameter p. What value of p maximizes $P(X = k)$?)