18.440 Problem Set 3

Due in class Wednesday September 24; late work will not be accepted. You can discuss problems with anyone, but you should write solutions entirely on your own.

1. (20 points) Problem 37 on page 107 (drawing cards from half decks). (To clarify the problem slightly: the ace is drawn from the *first* half deck, and then placed in the second. The answer is in the back of the book; you need to explain how to get it.)

2. (30 points) A coin is flipped three times. Consider the two events

$$E = \text{the first two tosses are the same}$$

$$F = \text{the last two tosses are the same.}$$

a) Suppose the coin is fair. Show that E and F are independent.
b) Suppose the coin is not fair, giving heads with probability $1/2 + x$, for some x with $0 < x < 1/2$. Show that

$$P(F|E) > P(F),$$

so that E and F are not independent.
c) Explain in words why the answer in (b) makes sense. Why is the case $x = 1/2$ different again?

3. (25 points) Problem 28 on page 119. (The setting is explained on pages 102–103. As in Laplace's rule, you are to assume that the number $k + 1$ of coins in the box is large.)

4. (25 points) This is a question about the Tests for Rare Diseases problem discussed on page 72 of the text. In the population at large, one person in ten thousand is a nerd. There is a test for this condition. When applied to an actual nerd, this test gives a positive result 99.99% of the time. When applied to a non-nerd, the test gives a negative result 99.99% of the time.

a) Suppose that you take the nerd test, and get a positive result. What is the probability that you are a nerd?
b) Suppose that the nerd test is Applying to MIT. The answer to part (a) might suggest that lots of students here are not nerds. Are there assumptions built into the calculation you did in (a) that may not be justified?