18.440 Problem Set 2

Due in class Monday September 15; late work will not be accepted. You can discuss problems with anyone, but you should write solutions entirely on your own.

1. (20 points)

a) How many possible five-card hands can be made from a deck of 52 cards?
b) How many five-card hands contain no pairs?
c) Assuming that all hands are equally likely, what is the probability of drawing a five-card hand with no pairs?

2. (20 points) Problem number 56 on page 58 of the text (three spinners).

3. (25 points)

a) Problem 18 on page 60 (coin tosses when successive heads never appear).
b) Find (try searching the internet for "Fibonacci") a formula for f_n.
c) Show that if n is large, the probability that n coin tosses result in no successive heads is very close to

$$\frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{4}\right)^n \simeq 1.1708(.8090)^n.$$

d) For n running from 0 to 4, prepare an attractive little table showing the *exact* probability of no successive heads in n tosses, the *approximate* probability given in part (c) (to four decimal places), and the *error* in the approximation.

4. (35 points) The purpose of this course is to convince you that the definition of probability given in this chapter is reasonable. The purpose of this problem is to convince you that it is not. Take for sample space the set

$$S = \{1, 2, 3, \dots\}$$

of positive integers. If E is any subset of S, define

$$E_n = E \cap \{1, 2, \dots, n-1, n\}.$$

Obviously E_n is a set having between 0 and n elements. Define the "density of E" to be

$$P(E) = \lim_{n\to\infty} \frac{\#E_n}{n}$$

if this limit exists. As in the Technical Remark on page 31, we will ignore those sets E for which the limit does not exist as "not of practical interest." I have used the words "explain why" instead of "prove" to indicate that your discussions can be informal; but they ought to be convincing.

a) Suppose $E \subset S$ and $P(E)$ is defined. Explain why $0 \le P(E) \le 1$.
b) Suppose $E \subset S$ and $P(E)$ is defined. Explain why $P(E^c) = 1 - P(E)$.
c) Suppose E and F are subsets of S, $P(E)$ and $P(F)$ are defined, and $E \cap F = \emptyset$. Explain why $P(E \cup F) = P(E) + P(F)$.
d) Let E be the set of positive integers divisible by 3. Explain why $P(E) = 1/3$.
e) Suppose F is a finite set of positive integers. Explain why $P(F) = 0$.
f) Let E be the set of positive integers whose base 10 expression has highest digit 1; that is, numbers like 17 and $1,026$. Explain why $P(E)$ is not defined.
g) Explain why (S, P) is not a probability space.