18.100B Problem Set 3

Due in class Monday, February 25. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 44, number 12.

2. Text, page 44, number 17.

A metric space X is said to be *totally bounded* if for every positive real number r, X is contained in a finite union of neighborhoods of radius r. That is, (for every $r > 0$) there are finitely many points $p_1, \ldots, p_n$ in X so that

$$X = N_r(p_1) \cup \cdots \cup N_r(p_n).$$

3.

a) Suppose that X is totally bounded and that $Y \subset X$. Show that Y is totally bounded.

b) Show that the unit interval $[0, 1]$ (with its usual metric) is totally bounded.

4. This problem concerns a new metric on the set $\mathbb{Z}$ of integers. If m and n are integers, define

$$d_2(m, n) = \begin{cases} 0, & \text{if } m = n \\ 2^{-a}, & \text{if } m \neq n \text{ and } 2^a \text{ is the largest power of 2 dividing } m - n. \end{cases}$$

(This is called the 2-*adic distance from m to n*.) For example, $d_2(20, 44) = 1/8$.

a) Show that d_2 is a metric on $\mathbb{Z}$.

b) Show that $\mathbb{Z}$ is totally bounded in the metric d_2.