18.05 Problem Set 7, Spring 2018

(due outside 4-174 Wednesday, Apr. 18 at 9:30 AM)

Problem 1. (10 pts.) **Spinning gold** When spun on edge 250 times a certain coin came up heads 140 times and tails 110. We can make the statement: 'if the coin is unbiased then the probability of getting a result at least this extreme is 7%.'

(a) Let θ be the probability of coming up heads. Consider the null hypothesis that the coin is fair, $H_0 : \theta = 0.5$. Carefully explain how the 7% figure arises. What term describes this value in NHST? Does it correspond to a one-sided or two-sided test?

(b) Would you reject H_0 at a significance level of $\alpha = 0.1$? What about $\alpha = 0.05$? (For this problem assume the test has the same sidedness as the one used to get the 7% value in part (a).)

(c) How many heads would you need to have observed out of 250 spins to reject at a significance of $\alpha = 0.01$? (Again assume the same sidedness as the test used to get the 7% value in part (a).)

(d) (i) Fix the significance level at $\alpha = 0.05$. Compute and compare the power of the test for values of the alternative hypothesis $\theta = 0.55$ and $\theta = 0.6$? (Again assume the same sidedeness as in part (a).)

(ii) Sketch the pmf of each hypothesis and use it to explain the change in power observed in part (i).

(e) (i) Again fix $\alpha = 0.05$. What is the *smallest* number of spins necessary for the test to have a power of 0.9 when $H_A : \theta = 0.55$? (Use the sidedness from part (a).)

(ii) As in part (d), use sketches to illustrate and explain the change in power.

(f) Let $H_A : \theta = 0.55$. Suppose we have only two hypotheses H_0 and H_A, and a flat prior $P(H_0) = P(H_A) = 0.5$. What is the posterior probability of H_A given the data? (In this part H_A is different from in the previous parts; it consists of one specific value of θ.)

Problem 2. (10 pts.) **Polygraph analogy.**
In an experiment on the accuracy of polygraph tests, 140 people were instructed to tell the truth and 140 people were instructed to lie. Testers use a polygraph to guess whether or not each person is lying. By analogy, let's say H_0 corresponds to the testee telling the truth and H_A corresponds to the testee lying.

(a) Describe the meaning of type I and type II errors in this context, and estimate their probabilities based on the table.

	Testee is truthful	Testee is lying
Tester thinks testee is truthful	131	15
Tester thinks testee is lying	9	125

(b) In NHST, what relationships exist between the terms significance level, power, type I error, and type II error?

Problem 3. (10 pts.) We perform a t-test for the null hypothesis $H_0 : \mu = 10$ at significance level $\alpha = 0.05$ by means of a dataset consisting of $n = 16$ elements with sample mean 11 and sample variance 4.

(a) Should we reject the null hypothesis in favor of $H_A : \mu \neq 10$?

(b) What if we test against $H'_A : \mu > 10$?

Problem 4. (10 pts.) (**z-test**) Suppose three radar guns are set up along a stretch of road to catch people driving over the speed limit of 40 miles per hour. Each radar gun is known to have a normal measurement error modeled on $N(0, 5^2)$. For a passing car, let $\bar{x}$ be the average of the three readings. Our default assumption for a car is that it is not speeding.

(a) Describe the above story in the context of NHST. Are the most natural null and alternative hypotheses simple or compound?

(b) The police would like to set a threshold on $\bar{x}$ for issuing tickets so that no more than 4% of the tickets are given in error.

(i) Use the NHST description in part (a) to help determine what threshold should they set.

(ii) Sketch a graph illustrating your reasoning in part (i).

(c) What is the power of this test with the alternative hypothesis that the car is traveling at 45 miles per hour? How many cameras are needed to achieve a power of 0.9 with $\alpha = 0.04$?

Problem 5. (10 pts.) One generates a number x from a uniform distribution on the interval $[0, \theta]$. One decides to test $H_0 : \theta = 2$ against $H_A : \theta \neq 2$ by rejecting H_0 if $x \leq 0.1$ or $x \geq 1.9$.

(a) Compute the probability of a type I error.

(b) Compute the probability of a type II error if the true value of θ is 2.5.