18.05 Problem Set 1, Spring 2018

(due outside 4-174 Monday, Feb. 12 at 9:30 AM)

Problem 1. (15 pts.) **Poker hands**.
(a) Some other five card poker hands are a straight and a flush

Straight: The ranks are in sequence, but not all the suits are the same. e.g. $\{7\heartsuit, 8\spadesuit, 9\clubsuit, 10\clubsuit, J\diamondsuit\}$. Note: the ace can be either high or low, that is 1,2,3,4,5 and 10,J,Q,K,A are both straights.

Flush: All five cards are of the same suit, e.g. $\{2\heartsuit, 3\heartsuit, 5\heartsuit, 10\heartsuit, K\heartsuit\}$

Four-flush: Some dishonest poker players try do what's known as four-flushing. When they have 4 cards of one suit and a fifth card of a different suit, but the same color they lay down their cards and declare it to be a flush. They hope that no one will notice that the fifth card is of the wrong suit. For example, $\{2\spadesuit, 3\spadesuit, 5\clubsuit, 10\spadesuit, K\spadesuit\}$

Calculate the probability of a straight and of a four-flush. Which is higher probability?

(b) Suppose the deck is shuffled and the cards are turned over one at a time.

(i) What is the probability the first spade turned over happens on the seventh card?

(ii) What is the probability that the last card turned over is a diamond?

Problem 2. (20 pts.) **A roll of the dice.**
(a) If a twelve-sided die is rolled against an eight-sided die with the higher number winning, what is the probability that the twelve-sided die wins?

What is the probability the twelve sided die wins if they play three rounds and the best two out of three is the winner?

(b) Suppose you have two pair of dice. The first pair is 2 six-sided dice and the second is a four-sided die and an eight-sided die. If the pairs are rolled against each other with the highest total winning, what is the probability the pair of six-sided dice wins?

(c) I have a set of dice containing a 4, 6, 8, and 12 sided die. Suppose I roll the set of four dice against a single 20-sided die, with the biggest total winning. Part (b) stretched my willingness to compute by hand to the limit. Use R to simulate the probability that the set of four dice wins. Set the number of trials to 10,000.

Problem 3. (20 pts.) **Sharing presents**.
Ignoring leap days, the days of the year can be numbered 1 to 365. Assume that birthdays are equally likely to fall on any day of the year. Consider a group of n people, of which you are not a member. An element of the sample space Ω will be a sequence of n birthdays (one for each person).

(a) Define the probability function P for Ω.

(b) Let A be the event that someone in the group shares *your* birthday.

Carefully describe the subset of Ω that corresponds to A and then find an exact formula for $P(A)$.

(c) Let B be the event that at least one pair of people in the group share a birthday. Find an exact formula for $P(B)$

(d) Is there a number of people n so that the probability $P(A)$ is guaranteed to be 1? What about $P(B)$. (*You need to explain your answers.*)