Probability Cast

Introduced so far

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes $S$ (or $\Omega$).
- Event: a subset of the sample space.
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in S$
  1. Probability is between 0 and 1
  2. Total probability of all possible outcomes is 1.
Example (from the reading)

Experiment: toss a fair coin, report heads or tails.

Sample space: \( \Omega = \{H, T\} \).

Probability function: \( P(H) = .5, \quad P(T) = .5 \).

Use tables:

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>H</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

(Tables can really help in complicated examples)
Discrete sample space

Discrete = listable

Examples:

\{a, b, c, d\} (finite)

\{0, 1, 2, \ldots\} (infinite)

\{sun, cloud, rain, snow, fog\}

\{patient cured, unchanged, patient died\}
Events

Events are sets:
- Can describe in words
- Can describe in notation
- Can describe with Venn diagrams

Experiment: toss a coin 3 times.

Event:
You get 2 or more heads = \{ \text{HHH, HHT, HTH, THH} \}
Events, sets and words

Experiment: toss a coin 3 times.

Which of following equals the event “exactly two heads”?

\[ A = \{ THH, HTH, HHT, HHH \} \]
\[ B = \{ THH, HTH, HHT \} \]
\[ C = \{ HTH, THH \} \]

(1) A (2) B (3) C (4) A or B
Experiment: toss a coin 3 times.

Which of the following describes the event \{THH, HTH, HHT\}?

(1) “exactly one head”
(2) “exactly one tail”
(3) “at most one tail”
(4) none of the above
Events, sets and words

Experiment: toss a coin 3 times.

The events “exactly 2 heads” and “exactly 2 tails” are disjoint.

(1) True      (2) False
Events, sets and words

Experiment: toss a coin 3 times.

The event “at least 2 heads” implies the event “exactly two heads”.

(1) True    (2) False
Sample space: $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$

Outcome: $\omega \in S$

Probability between 0 and 1:

Total probability is 1:

Event $A$: $P(A)$
Probability and set operations on events

Events $A$, $L$, $R$

- Complements: $P(A^c) = 1 - P(A)$.
- Disjoint events: If $L$ and $R$ are disjoint then $P(L \cup R) = P(L) + P(R)$.
- Inclusion-exclusion principle: For any $L$ and $R$: $P(L \cup R) = P(L) + P(R) - P(L \cap R)$.

$\Omega = A \cup A^c$, no overlap

$L \cup R$, no overlap
Class has 50 students
20 male (M), 25 brown-eyed (B)

What is the range of possible values for $p = P(M \cup B)$?

(a) $p \leq .4$
(b) $.4 \leq p \leq .5$
(c) $.4 \leq p \leq .9$
(d) $.5 \leq p \leq .9$
(e) $.5 \leq p$
Table Question

Experiment:

1. Your table should make 9 rolls of a 20-sided die (one each if the table is full).

2. Check if two rolls at your table match.

Repeat the experiment five times and record the results.
Table Question

Experiment:

1. Your table should make 9 rolls of a 20-sided die (one each if the table is full).

2. Check if two rolls at your table match.

Repeat the experiment five times and record the results.

For this experiment, how would you define the sample space, probability function, and event?

Compute the true probability that two rolls (in one trial) match and compare with your experimental result.
Jon’s dice

Jon has three six-sided dice with unusual numbering.

A game consists of two players each choosing a die. They roll once and the highest number wins.

Which die would you choose?
Board Question

1. Make probability tables for the red and white dice.
2. Make a probability table for the product sample space of red and white.
3. Compute the probability that red beats white.

4. Pair up with another group. Have one group compare red vs. green and the other compare green vs. white. Based on the three comparisons rank the dice from best to worst.
The 2 × 2 tables show pairs of dice.
Each entry is the probability of seeing the pair of numbers corresponding to that entry.
The color gives the winning die for that pair of numbers. (We use black instead of white when the white die wins.)
Answer to board question continued

<table>
<thead>
<tr>
<th></th>
<th>White</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 5</td>
<td>1 4</td>
</tr>
<tr>
<td>Red</td>
<td>3</td>
<td>15/36 15/36</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3/36 3/36</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
<td>3/36 3/36</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>15/36 15/36</td>
</tr>
</tbody>
</table>

The three comparisons are:

\[
P(\text{red beats white}) = \frac{21}{36} = \frac{7}{12}\\
P(\text{white beats green}) = \frac{21}{36} = \frac{7}{12}\\
P(\text{green beats red}) = \frac{25}{36}
\]

Thus: red is better than white is better than green is better than red.

There is no best die: the relation ‘better than’ is not transitive.
Concept Question

Lucky Larry has a coin that you’re quite sure is not fair.

- He will flip the coin twice
- It’s your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same
2. Different
3. It doesn’t matter, same and different are equally likely
Board Question

Lucky Larry has a coin that you’re quite sure is not fair.

- He will flip the coin twice
- It’s your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?
1. Same 2. Different 3. Doesn’t matter

**Question:** Let $p$ be the probability of heads and use probability to answer the question.

(If you don’t see the symbolic algebra try $p = .2$, $p = .5$)