April 1, 2011

18.01 Problem Set 9
Due Wednesday, April 13, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes.

1. 4J-1, 3, 6.
2. 5B-5,9, 14; 5C-3, 5, 6, 14.

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."

1a) Find the average value of the function $1 = \cos^0(x)$ on the interval $0 \le x \le 2\pi$.

b) Find the average value of the function $\cos^2(x)$ on the interval $0 \le x \le 2\pi$. (You can just quote an answer from the last problem set if you prefer.)

c) Find the average value of the function $\cos^4(x)$ on the interval $0 \le x \le 2\pi$.

d) Find the average value of the function $\cos^6(x)$ on the interval $0 \le x \le 2\pi$.

e) Explain why the answers for (a)–(d) are decreasing.

2) You can use the formulas

$$\cos(ax)\cos(bx) = \frac{1}{2}\left(\cos((a+b)x) + \cos((a-b)x)\right)$$

$$\cos(ax)\sin(bx) = \frac{1}{2}\left(\sin((a+b)x) + \sin((a-b)x)\right)$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\left(-\cos((a+b)x) + \cos((a-b)x)\right)$$

to write things like $\sin^M(x)\cos^N(x)$ as sums of terms like $\cos(nx)$ and $\sin(nx)$ (with n smaller than $M+N$).

a) Use this idea to find a formula of the form

$$\cos^3(x) = A\cos(3x) + B\cos(2x) + C\cos(x) + D.$$

b) Use the formula in (a) to calculate

$$\int \cos^3(x)dx.$$

c) Use this idea to say as much as you can about the trigonometric identity

$$\cos^n(x) = a_n\cos(nx) + a_{n-1}\cos((n-1)x) + a_{n-2}\cos((n-2)x) + \cdots + a_1\cos(x) + a_0.$$

(Best answer is a formula for every coefficient a_n. But if you can say something like, "the last term a_0 is 11 when n is odd," or "every fifth term is zero," that's good too.)