

February 24, 2011

18.01 Problem Set 4
Due Wednesday, March 2, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

The point of Part I is to push you to do some of the routine practice necessary to master this material. In order for that to work, you should write your solutions without consulting the solutions provided. There's no value for anyone in faithfully transcribing the typographical errors in those printed solutions.

Some of the questions in Part II are vague and open-ended, with no clear right answer. That's intentional; it's more like the form in which real mathematical questions appear. Say as much as you can.

Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes (solved in section S).

1. 2B-(1 and 2)ad
2. 2C-1, 6, 7
3. 2E-1, 4, 5

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."
- 1a) Do problem 2A-5 from the supplementary notes.
- b) Look on the web to find some kind of table describing average weights as a function of height. (It's easy to find some old "ideal weight" tables like this, and one of those would be OK; it would be more interesting to find tables recording averages of measurements of real people.) Write the URL where you found the table, and the weights in the table associated to heights of 60 and 61 inches. (If the table you found doesn't include those heights, or is calibrated in centimeters and kilograms, then you should also repeat the calculation in 1(a) to guess the number for the larger height from the number for the smaller height.)
- c) Is the data you found compatible with the result of 2A-5 (also available in the supplementary notes)? Can you offer an explanation for any difference?
- 2a) Three chess players A , B , and C live on the x axis, at coordinates 0, 1, and 20. They plan to hold a chess club meeting at some coordinate x along the axis (to be chosen). How should they choose x so as to minimize the total distance they need to travel to get to the meeting?
- b) Same as (a), but now the coordinates of the chess players are just given as three numbers a , b , and c ; and your answer should be expressed in terms of a , b , and c .
- c) Same as (b), but now minimize the sum of the *squares* of the travel distances for the three chess players.

3. The N students in a class take an exam, receiving scores of a_1, a_2, \dots, a_N . The professor wishes to report to the class some number x that reflects a typical score.

a) Suppose that the professor wants to report a number that minimizes the sum of the absolute values of the differences from actual scores. This means that if he reports x , he wants to minimize

$$M(x) = |x - a_1| + |x - a_2| + \cdots + |x - a_N|.$$

What value of x minimizes $M(x)$? (Hints: this problem is more or less the same as Problem 2, except that 3 has become N . Solving it by calculus is hard, because the absolute value function doesn't have a derivative at 0. The answer to this question is not difficult or deep, but finding that answer is not something you "should" be able to do.)

b) Suppose that the professor wants to report a number that minimizes the sum of the squares of the differences from actual scores. This means that if he reports x , he wants to minimize

$$A(x) = (x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_N)^2.$$

What value of x minimizes $A(x)$?

4. This problem is about the function $\cos(x)$ and its quadratic approximation at zero $Q(x) = 1 - x^2/2$.

a) Sketch on the same axes the graphs of \cos and of Q in the range $-\pi \leq x \leq \pi$.

b) Is it true that $\cos(x) \geq Q(x)$ for all x ?