18.01 Problem Set 4.5 UNASSIGNED PRACTICE

Since I neglected to prepare a problem set due on March 9, these problems are intended to offer similar value in preparing for the exam on Friday March 11. THEY ARE NOT TO BE HANDED IN!

As always, the problems in the supplementary notes are excellent ones for test preparation; the Review Problems at the end of the notes are a good beginning.

Part I

- 1. 2E-3, 8
- 2. 2F-1, 4.additional/119c
- 3. 2G-1ab, 3, 2.6/40

Part II

1. This problem concerns the function $f(x) = \sqrt{1 - x^2}$ on the interval $0 \le x \le 1$. I want to approximate this slightly complicated function by a straight line y = ax + b, where a and b are to be chosen. Usually when we talk about linear approximations in calculus, we are interested in x close to one special value x_0 ; but here I am considering all values of x equally. Since the function f(x) is strictly decreasing, it's reasonable to think that a good approximation should be decreasing as well; so I will assume

a is strictly negative.

- a) Show that f is concave down on the interval 0 < x < 1.
- b) Show that the derivative f'(x) decreases from 0 toward $-\infty$ as x increases from 0 to 1.
- c) Show that the "error function"

$$e(x) = f(x) - ax - b$$

has exactly one critical point between 0 and 1, and that this critical point is a maximum.

- d) Show that the value of e at the critical point is $\sqrt{1+a^2}-b$.
- e) One way to choose a "best" straight line approximation to f is to require that the maximum value of |e(x)| be as small as possible. Explain why this maximum value is the largest of the three numbers

$$|1-b|, |\sqrt{1+a^2}-b|, |-a-b|.$$

- f) Show that if $-1 \le a \le 0$, then the maximum value of |e| is $(\sqrt{1+a^2}+a)/2$, and find a similar formula for the case $-\infty < a \le -1$.
- g) What is the best straight line approximation to f? What is the maximum error of this approximation?
- 2. (This is almost problem 31 on page 138 of Simmons.) A spider is in a corner of the ceiling in a room of length a, width b, and height h; now $a \ge b \ge h$. The spider wishes to catch a bug at the opposite corner of the floor of the room; so it could reach the bug by walking along a long edge of the ceiling (distance a), then a short edge of the ceiling (distance b), then down one corner of the room (distance h), for a total distance a + b + h. But it can do better than this. Find the length of the spider's shortest possible path (walking along ceilings and walls and floors) to the bug. (It's possible to do this without calculus by being clever; that's acceptable, but calculus is OK too.)