February 17, 2011

**18.01 Problem Set 3**
**Due Wednesday, February 23, in recitation**

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

The point of Part I is to push you to do some of the routine practice necessary to master this material. In order for that to work, you should write your solutions without consulting the solutions provided. There's no value for anyone in faithfully transcribing the typographical errors in those printed solutions.

Some of the questions in Part II are vague and open-ended, with no clear right answer. That's intentional; it's more like the form in which real mathematical questions appear. Say as much as you can.

## Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes (solved in section S).

1. 2A-1, 4, 12cd
2. 2A-14

## Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."

1. This problem is about solving equations like

$$f(x) = x$$

for some function $f$. The idea is that the equation may be too complicated to solve algebraically, so you look for a numerical approximation.

a) Get a calculator that can calculate cos in radians. Making sure that the calculator is set for radians, enter .1, then hit the cosine key repeatedly, at least fifteen or twenty times. Say what you see. (For example, you might say, "the numbers are getting closer and closer to zero; every time I hit the key, twice as many decimal places are zero.")

b) Find an approximate solution $x_0$ to the equation $\cos(x) = x$. Say how accurate you believe your solution is, and why.

c) Repeat part (a) (again starting with .1) this time hitting the sine key repeatedly.

d) Find an exact solution $x_0$ to the equation $\sin(x) = x$.

2. This problem aims to understand what you saw in the first problem. Suppose $f$ is a differentiable function of $x$, and that there is a solution $x_0$ to the equation

$$f(x) = x.$$

a) Suppose $x$ is an approximation to $x_0$, with an error of $(x - x_0)$. Using a linear approximation to $f$ at $x_0$, find an approximation for $f(x) - x_0$.

b) Under what conditions would you expect that $f(x)$ is a *better* approximation to $x_0$ than $x$?

c) In the case of the equation $\cos(x) = x$, suppose you start with an approximation $x$ having an error of about .1. About how many times should you hit the cosine key to get an approximation with an error of about $10^{-6}$?