February 11, 2010

18.01 Problem Set 2
Due Wednesday, February 16, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

Some of the questions in Part II are open-ended, with no clear right answer. That's intentional; it's more like the form in which real mathematical questions appear. Say what you can.

Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes (solved in section S).

5. 1F-3, 5, 8bc; 5A-1adef, 5A-3ag
6. 1H-1, 1H-2, 1H-4, 1H-5a; 1I-1cdh, 4b.

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."

1a) For n a positive integer, calculate the derivative of the function

$$f_n(x) = n[x^{1/n} - 1] \qquad (x > 0).$$

b) Calculate $\lim_{n\to\infty} f_n'(x)$. (This is meant to be very easy.)

c) As n gets bigger and bigger, the derivative of f_n is getting closer and closer to the derivative of ln. You might suspect that f_n is getting closer and closer to ln. To check whether this is reasonable, use a calculator to compute $f_{10}(2)$, $f_{100}(2)$, and $\ln(2)$.

d) Suppose $n = 2^k$ is a power of 2. Explain how to calculate calculate f_n by repeated extraction of square roots (together with subtraction and multiplication by 2). Could this be a reasonable way for computers to calculate natural logarithms?

2. This problem is about the circle $x^2 + y^2 = 1$.

a) Use implicit differentiation to calculate the slope of the tangent line to the circle at (x, y).

b) Calculate the slope of the line through the origin and the point (x, y).

c) Two lines L_1 and L_2 are perpendicular to each other if their slopes m_1 and m_2 are negative reciprocals; that is, $m_1 m_2 = -1$. Can you find a geometric reason for the lines in (a) and (b) to be perpendicular to each other?

3a) The derivative of the function $2u^2 - 1$ with respect to u is $4u$. Use this fact and the chain rule to calculate the derivative of $2\cos^2(\theta) - 1$ with respect to θ.

b) Calculate the derivative of $1 - 2\sin^2(\theta)$ with respect to θ. What does this have to do with (a)?

c) A *trigonometric polynomial* is a sum of terms of the form $a\cos^m(\theta)\sin^n(\theta)$. Explain why the derivative of any trigonometric polynomial is another trigonometric polynomial.

d) It's not hard to see that any polynomial is the derivative of another polynomial. Is this true for *trigonometric* polynomials?