April 15, 2011

## 18.01 Problem Set 12
## Due Wednesday, May 4, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

### Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes.

1. 4H-1ag, 2ad.

2. 6A-1bcf, 6A-2egi.

### Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."

1a) (5 points) Remember that the hyperbolic sine and cosine are the functions defined by

$$\sinh x = (e^x - e^{-x})/2, \qquad \cosh x = (e^x + e^{-x})/2.$$

Prove the formulas
$$\cosh^2 x - \sinh^2 x = 1, \qquad \frac{d \sinh x}{dx} = \cosh x.$$

b) Sketch the graph of $\sinh x$, and explain why the inverse function $\sinh^{-1}(x)$ is defined for all values of $x$.

c) Show that the derivative of the inverse hyperbolic sine is given by the formula

$$\frac{d \sinh^{-1} x}{dx} = 1/\sqrt{1 + x^2}.$$

d) Show that
$$\int_0^x \frac{dt}{\sqrt{1 + t^2}} = \sinh^{-1} x.$$

2. (5 points) Evaluate
$$\int \frac{dt}{\sqrt{1 + t^2}}$$

using the trigonometric substitution $t = \tan \theta$. Your final answer will involve logarithms and square roots.

3. (5 points) Solve the equation
$$2y = e^x - e^{-x}$$

for $x$.