April 15, 2011

18.01 Problem Set 11
Due Wednesday, April 27, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

Part I: 10 points

Notation for homework problems: "2.4/13" means Problem 13 at the end of section 2.4 in Simmons. "1A-3" means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes.

1. 5F-1, 2dc, 3, 4.
2. 4E-1, 3, 7; 4F-1b, 7, 8; 4G-1, 3, 7.

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say "none" or "no consultation."

1. (10 points) This problem is about the functions $x^m e^{-x}$, with m a non-negative integer.

1a) Calculate the average value A_0 of e^{-x} over the interval $[0, 1]$.

b) Calculate the average value A_1 of xe^{-x} over the interval $[0, 1]$.

c) Calculate the average value A_2 of x^2e^{-x} over the interval $[0, 1]$.

d) Prove a reduction formula of the form

$$\int x^m e^{-x} dx = C_m x^m e^{-x} + D_m \int x^{m-1} e^{-x} dx.$$

e) Explain how to calculate the average value A_m of $x^m e^{-x}$ over $[0, 1]$ from A_{m-1}.

f) Show that there are integers a_m and b_m with the property that

$$A_m = a_m - \frac{b_m}{e}.$$

Explain how to calculate a_m and b_m from a_{m-1} and b_{m-1}.

g) Explain why A_m is between $\frac{1}{(m+1)}$ and $\frac{1}{e(m+1)}$.

2. (5 points) Explain why $e - (1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{m!})$ is between $\frac{1}{(m+1)!}$ and $\frac{e}{(m+1)!}$. (This means, for instance, that the error in the approximation

$$e \approx 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{15!}$$

is at most $e/15! \approx 2 \times 10^{-12}$.)