

February 3, 2010

**18.01 Problem Set 1**  
**Due Wednesday, February 9, in recitation**

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

**Part I: 10 points**

Notation for homework problems: “2.4/13” means Problem 13 at the end of section 2.4 in Simmons. “1A-3” means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes (solved in section S).

0. 1A-2a, 3ad, 6b.
1. 1B-1abc, 1C-3bd, 1C-4bd, 1C-5.
2. 1D-1bfj, 3ab, 5ab, 6a.
3. 1E-1abc, 3, 4a, 5bc, 1J-1e, 1J-2.
4. 1F-1ab, 1F-4, 1F-7bd, 1J-1ak, 1G-4.

**Part II: 15 points**

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say “none” or “no consultation.”
- 1a) A circle of radius  $r$  has area  $\pi r^2$  and circumference  $2\pi r$ . The rate of change of the area with respect to the radius is therefore equal to the circumference. Explain geometrically why this is so.
- b) What is the analogous fact for squares?
2. The *greatest integer in  $x$* , written  $[x]$ , is defined to be the largest integer that is less than or equal to  $x$ . Thus for example

$$[2.5] = 2, \quad [\pi] = 3, \quad [10] = 10, \quad [-2/3] = -1.$$

The *fractional part of  $x$* , written  $\{x\}$ , is defined by

$$\{x\} = x - [x].$$

For example,

$$\{2.5\} = .5, \quad \{\pi\} = .1415926535\dots, \quad \{10\} = 0, \quad \{-2/3\} = 1/3.$$

- a) Graph the function  $h(x) = \{x\} - \{x\}^2$  on the interval from  $-2$  to  $2$ .
- b) Is the function  $h(x)$  continuous? Differentiable? Periodic? Even? Odd? Explain why or why not.
- 3a) Graph (together) the three functions  $f(x) = x$ ,  $g(x) = -x$ , and  $h(x) = x \cos(x)$  on the interval from  $x = -\pi$  to  $x = \pi$ .
- b) Find the slope of the tangent line to the graph of  $h$  at  $x = \pi/4$ .
- c) When  $x = \pi/4$ ,  $h(x)$  takes the value  $\pi\sqrt{2}/8$  (about .555). Is that the largest value it takes for  $x$  between  $-\pi$  and  $\pi$ ? (Hint: you don't need to do any additional calculation.)