

Harvard-M.I.T. Algebraic Geometry Seminar

CLASS NUMBERS OF ALGEBRAIC GROUPS OVER FUNCTION FIELDS

BRIAN CONRAD

University of Michigan and Columbia

For any algebraic group over a global field one can use adelic double cosets to define a concept of "class number" that generalizes the notion of generalized ideal class numbers for number fields (in the case of the multiplicative group). It is a theorem of Borel from the 1960's that algebraic groups over number fields have finite class numbers; his proof used fundamental domains. Over global function fields there arise numerous new problems caused by both inseparable isogenies and especially subtleties with unipotent groups over imperfect fields. We show some interesting examples to illustrate the new complications and sketch an entirely different method to handle these problems and to thereby prove finiteness in many cases, and essentially all cases assuming an interesting conjecture of Gabber (on which we have made significant progress). In the number field case this bypasses Borel's work with fundamental domains. The key arithmetic inputs are cohomological results of Kneser-Tits and Harder for simply connected groups over local and global function fields.

Tuesday, February 27th

3:00 p.m.

MIT Room 4-145

<http://www-math.mit.edu/ags/>