

THE BOUNDED BELOW EQUIVARIANT DERIVED CATEGORY

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ABSTRACT. In this note, we give show how the equivariant derived category of Bernstein and Lunts can be extended to a purely algebraic setting, without the use of the theory of stacks. In particular, we show that it is possible to use the yoga of weights in the equivariant derived category, as one would in the usual derived category of a scheme over a finite field.

1. Introduction. In this note, we define the equivariant derived category $D_G^+(X)$ for certain varieties X acted on by an algebraic group G . This category is intended to play the role of the derived category of the stack X/G , but can be defined without using the full power of the formalism of stacks. One version of this was done by Bernstein and Lunts [BL94] (the basic properties of their formalism are summarized more concisely in our previous paper [WW08]), and our description will spring from the same basic ideas as theirs. However, their work is essentially topological in nature. We require a more general definition, which will allow us to also work with varieties over characteristic p , and thus obtain a mixed version of the equivariant derived category.

As we noted above, this category was first constructed (in the topological setting) by Bernstein and Lunts [BL94]. In [Sch] one finds a detailed description of this category for certain complex varieties. We need to recall this construction, as its finer points are necessary for the proofs of invariance later on.

We also need a definition of the bounded below category $D_G^+(X)$ (as opposed to $D_G^b(X)$), which is not trivial to obtain. This case is discussed in Bernstein-Lunts in the topological situation with the help of an ∞ -acyclic resolution. However such a resolution is not available in the category of algebraic varieties. Our approach is to observe that the definition in [Sch] as an inverse limit of categories generalizes to our situation.)

As shown in Chapter 18 of [LMB00], $D_G^b(X)$, (resp. $D_G^+(X)$) is the full subcategory of “cartesian sheaves” in the bounded (resp.

bounded below) derived category of constructible sheaves on the stack $[X/G]$. The reader comfortable with the language of stacks can probably ignore this note and use this as their definition of $D_G^+(X)$, but as working mathematicians, we find it more congenial to have a more “hands-on” definition in this special case. The papers [LO08a], [LO08b] and [LO] provide an excellent reference for the theory of constructible sheaves on stacks and proves the existence of a “Grothendieck formalism” in a general setting.

2. Inverse limit categories. Let (I, \leq) be a partially ordered set and suppose that we are given:

- (1) categories \mathcal{C}_i for each $i \in I$;
- (2) functors $F_{ij} : \mathcal{C}_j \rightarrow \mathcal{C}_i$ whenever $i \leq j$ such that
 - (a) $F_{ii} = id$ for all $i \in I$,
 - (b) $F_{ij}F_{jk} = F_{ik}$ if $i \leq j \leq k$.

We will denote the effect of F_{ij} on an object M (resp. morphism f) in \mathcal{C}_j by $M \mapsto M|_{\mathcal{C}_i}$ and $f \mapsto f|_{\mathcal{C}_i}$.

Definition 1. *Given the above data, the **inverse limit category** is the category $\varprojlim \mathcal{C}_i$ defined as follows:*

- (1) *Its objects consist of pairs (M_i, ϕ_{ij}) where $M_i \in \mathcal{C}_i$ and ϕ_{ij} is an isomorphism*

$$\phi_{ij} : M_j|_{\mathcal{C}_i} \xrightarrow{\sim} M_i$$

such that $\phi_{ii} = id$ and $\phi_{ij}(\phi_{jk}|_{\mathcal{C}_i}) = \phi_{ik}$ for all $i \leq j \leq k$.

- (2) *A morphism between two objects (M_i, ϕ_{ij}) and (N_i, ψ_{ij}) is given by morphisms $\alpha_i : M_i \rightarrow N_i$ for all $i \in I$ such that, for all $i < j$ we have a commutative diagram:*

$$\begin{array}{ccc} M_j|_{\mathcal{C}_j} & \xrightarrow{\phi_{ij}} & M_i \\ \downarrow \alpha_j|_{\mathcal{C}_j} & & \downarrow \alpha_i \\ N_j|_{\mathcal{C}_j} & \xrightarrow{\psi_{ij}} & N_i. \end{array}$$

We will sometimes abuse notation and write simply $M = (M_i)$ for an object in $\varprojlim \mathcal{C}_i$.

Remark 1. As the notation suggests this data gives a “sheaf of categories on I ”. The inverse limit category may be thought of as the global sections of this sheaf.

By a **G -resolution** (or simply resolution) of a G -variety X we mean a smooth G -equivariant morphism $P \xrightarrow{p} X$ from a free G -variety P with quotient $\bar{P} := P/G$. We denote this situation by

$$X \xleftarrow{p} P \xrightarrow{\bar{p}} \bar{P}.$$

A morphism of resolutions is a G -equivariant map $P \rightarrow Q$ over X . We denote the category of G -resolutions of X by $\text{Res}_G(X)$.

Given a resolution $P \xrightarrow{p} X$ the category $D^+(\bar{P}|p)$ of **sheaves on \bar{P} coming from X** is the full subcategory of $D^+(\bar{P})$ of objects \mathcal{F} such that $\bar{p}^*\mathcal{F} \cong p^*\mathcal{G}$ for some $\mathcal{G} \in D^+(X)$. We say that a morphism $\alpha : \mathcal{F} \rightarrow \mathcal{F}'$ **comes from X** if there is a morphism $\beta : \mathcal{G} \rightarrow \mathcal{G}'$ of sheaves on X and a commutative diagram

$$\begin{array}{ccc} \bar{p}^*\mathcal{F} & \longrightarrow & \bar{p}^*\mathcal{F}' \\ \downarrow \sim & & \downarrow \sim \\ p^*\mathcal{G} & \longrightarrow & p^*\mathcal{G}' \end{array}$$

Note that it is not clear whether every morphism in $D^+(\bar{P}|p)$ comes from X and hence $D^+(\bar{P}|p)$ is not *a priori* a triangulated subcategory.

Recall that a map $f : X \rightarrow Y$ is called **n -acyclic** if, for every base change $f' : X' \rightarrow Y'$ of f and any sheaf \mathcal{F} concentrated in degree 0 on Y' the truncated adjunction map

$$\mathcal{F} \rightarrow \tau_{\leq n} f'_* f'^* \mathcal{F}$$

is an isomorphism. Note that if f is n -acyclic then it is m -acyclic for all $m \leq n$ and that 0-acyclic maps are surjective.

We call a subcategory $I \subset \text{Res}_G(X)$ **acyclic** if it has the following properties:

- (1) I is equivalent to a partially ordered set with upper bounds,
- (2) every $(P, p) \in I$ admits a morphism to an n -acyclic resolution of X , for all n .

The first condition means that all Hom spaces are either empty or contain one element, I does not contain any cycles and that any two resolutions in I admit maps in I to a third resolution. We will comment later on the existence of acyclic subcategories.

For all resolutions $(P, p) \in I$ we have categories $D^+(\bar{P}|p)$ and for any morphism $i : (P, p) \rightarrow (Q, q)$ we have a restriction functor

$$i^* : D^+(\bar{Q}|q) \rightarrow D^+(\bar{P}|p)$$

such that, for any two composable arrows $i, j \in I$ we have $i^*j^* = (j \circ i)^*$.

Definition 2. *The (bounded below) equivariant derived category is the inverse limit*

$$D_G^+(X) := \varprojlim_I D^+(\overline{P}|p).$$

The **bounded equivariant derived category** $D_G^b(X)$ is the full subcategory of $D_G^+(X)$ consisting of sheaves $\mathcal{F} = (\mathcal{F}_P) \in D_G^+(X)$ satisfying $\mathcal{F}_P \in D^b(\overline{P}|p)$ for all $(P, p) \in I$.

This definition is important in that it shows that any notion for sheaves which is preserved by pullback by smooth maps makes sense for the equivariant derived category. For example, we can define a perverse t -structure on $D_G^+(X)$ by using the perverse t -structure on $D^+(\overline{P}|p)$ (taking into account that the pullback by a smooth map shifts the perverse t -structure by the relative dimension).

Furthermore, if X is defined over an algebraically closed field of characteristic p , then we have “Frobenius-equivariant” version of the derived category, the **mixed derived category** $D_{mix}^+(X)$.

Definition 3. *The mixed equivariant derived category is the inverse limit*

$$D_{G,mix}^+(X) := \varprojlim_I D_{mix}^+(\overline{P}|p).$$

Thus, all familiar concepts from the yoga of weights (function-sheaf correspondence, weight filtration on perverse sheaves, etc.) carry over to the equivariant situation without any problem.

3. Independence and existence of resolutions and triangulated structure. There are two clearly important questions that need to be addressed in view of this definition: does it depend on the choice of resolution, and when can one find such a resolution?

Proposition 4. *For any affine algebraic group G over an algebraically closed field k , and any G -scheme X , there is an acyclic subcategory $I \subset \text{Res}_G X$.*

Proposition 5. *Up to equivalence $D_G^+(X)$ does not depend on the choice of acyclic subcategory $I \subset \text{Res}_G(X)$.*

Proof of Proposition 4. First, note that that we only prove that an acyclic subcategory I exists for the point, since for any other X , we have that $X \times I \subset \text{Res}_G X$ is also acyclic. Similarly, if we prove the result for G , that establishes it for any subgroup $H \subset G$, since a resolution will remain acyclic upon restriction of the group.

Since G is affine, one can always embed $G \hookrightarrow GL_\ell$ for some ℓ . This reduces the result to proving such a resolution exists for GL_ℓ .

In this case, the resolutions are given by the Stiefel variety $E_n G$ given by all linear inclusions $k^\ell \rightarrow k^n$ with the obvious GL_ℓ -action. It is standard that

- (1) the quotient map $E_n G \rightarrow E_n G/G$ exists and is a Zariski locally trivial principle fibre bundle;
- (2) the map $E_n G \rightarrow pt$ is n -acyclic.
- (3) one has G -equivariant closed immersions $E_n G \hookrightarrow E_{n+1} G$ for all n .

One may then take I to be the system $\{E_n G\}$ equipped with the maps induced by the closed embeddings $E_n G \hookrightarrow E_{n+1} G$. \square

Sketch of the proof of Proposition 5. Let $I, J \subset \text{Res}_G X$ be two acyclic subcategories. We denote by $I \times J$ the acyclic subcategory formed by taking fibre products over X . (Note that if $P \rightarrow X$ and $Q \rightarrow Y$ are (at least) n -acyclic over X then so is their fibre product.) It is clearly enough to show an equivalence

$$\varprojlim_{R \in I \times J} D^+(\overline{R}|r) \cong \varprojlim_{Q \in J} D^+(\overline{Q}|q).$$

Note also, that for every n -acyclic resolution $P \in I$ and resolution $Q \in J$ we have an n -acyclic map

$$\pi_{P,Q} : P \times_X Q \rightarrow Q.$$

Pullback along these maps defines a functor

$$L : \varprojlim_{Q \in J} D^+(\overline{Q}|q) \rightarrow \varprojlim_{R \in I \times J} D^+(\overline{R}|r).$$

We also have a functor

$$\begin{aligned} R : \varprojlim_{R \in I \times J} D^+(\overline{R}|r) &\rightarrow \varprojlim_{Q \in J} D^+(\overline{Q}|q) \\ (\mathcal{F}_{P,Q})_{(P,Q) \in I \times J} &\mapsto \left(\varprojlim_P \pi_{P,Q,*} \mathcal{F}_{P,Q} \right)_{Q \in J} \end{aligned}$$

In fact, (L, R) form an adjoint pair and the corresponding unit $id \rightarrow RL$ is an isomorphism (the second statement follows from n -acyclicity). Hence L is fully faithful. Lastly, L is essentially surjective on bounded sheaves (again by n -acyclicity), commutes with colimits, and every object \mathcal{F} in $\varprojlim_{R \in I \times J} D^+(\overline{R}|r)$ is a colimit of bounded objects. Hence L is essentially surjective. \square

4. Triangulated structure. Note that, as each $D^+(\overline{P}_n|p_n)$ is not necessarily triangulated, it is not obvious that $D_G^+(X)$ can be given the structure of a triangulated category. This will follow from the following lemma:

Lemma 6. *Let $\alpha : \mathcal{F} \rightarrow \mathcal{G}$ be a morphism in $D_G^+(X)$. Then for all resolutions $P \in I$ the morphism $\alpha_P : \mathcal{F}_P \rightarrow \mathcal{G}_P$ comes from X .*

Before proving the lemma, note that $\varprojlim_I D^+(\overline{P}_n)$ has an obvious structure as a triangulated category obtained by declaring the distinguished triangles to be those triangles

$$\mathcal{F} \rightarrow \mathcal{G} \rightarrow \mathcal{H} \xrightarrow{[1]}$$

for which $\mathcal{F}_P \rightarrow \mathcal{G}_P \rightarrow \mathcal{H}_P \xrightarrow{[1]}$ is a distinguished triangle for all resolutions $P \in I$ (check that there is no subtlety here). However the above lemma implies that if \mathcal{F} and \mathcal{G} belong to $\varprojlim D^+(\overline{P}_n|p_n)$ then so does \mathcal{H} . Thus $D_G^+(X)$ inherits the structure of a triangulated category.

Proof. Observe that if the lemma is true for $P \in I$ it is true for all resolutions admitting a morphism to P in I . Observe also that if \mathcal{F} and \mathcal{G} are in $D_G^{[0,n]}(X)$ then, for all n -acyclic resolutions $P \xrightarrow{p} X$,

$$p^* \tau_{\leq n} p_* \overline{p}^* \mathcal{F}_P \xrightarrow{\sim} \overline{p}^* \mathcal{F}_P$$

and hence α_P is isomorphic to an arrow coming from X (namely $\tau_{\leq n} p_* \overline{p}^* \alpha_P$). Now fix a resolution P . The above arguments show that we can form a commutative diagram

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \tau_{\leq \ell} \mathcal{F}_P & \longrightarrow & \tau_{\leq \ell} \mathcal{G}_P \\ \downarrow & & \downarrow \\ \tau_{\leq \ell+1} \mathcal{F}_P & \longrightarrow & \tau_{\leq \ell+1} \mathcal{G}_P \\ \downarrow & & \downarrow \end{array}$$

with horizontal arrows coming from X and vertical arrows the natural morphisms. Taking the colimit shows that $\alpha_P : \mathcal{F}_P \rightarrow \mathcal{G}_P$ also comes from X . (Again it is important that \overline{p}^* and p^* commute with colimits). \square

5. Functoriality. We now discuss various functors between equivariant derived categories. Let $f : X \rightarrow Y$ be a morphism of G -varieties and $I \subset \text{Res}_G(Y)$ be an acyclic subcategory. Base change along f yields an acyclic subcategory $f^*I \subset \text{Res}_G(X)$. Using I and f^*I to define $D_G^+(Y)$ and $D_G^+(X)$ respectively one obtains functors

$$\begin{array}{ccc} & \xrightarrow{f_*, f!} & \\ D_G^+(X) & & D_G^+(Y) \\ & \xleftarrow{f^*, f!} & \end{array}$$

obeying a Grothendieck formalism. For example, if $\mathcal{F} = \{\mathcal{F}_{f^*P}\} \in D_G^+(X)$ then we have

$$f_*\mathcal{F} = \{\overline{f_P} \mathcal{F}_{f^*P}\}$$

where, for a resolution $P \xrightarrow{p} X$, $\overline{f_P} : \overline{f^*P} \rightarrow \overline{P}$ is the induced map. These functors preserve the full subcategories of bounded sheaves and commute with the forgetful functor to be defined below.

We now describe functors of induction and restriction. Suppose that $H \subset G$ is a closed subgroup. Any n -acyclic H -resolution $P \rightarrow X$ gives an n -acyclic G -resolution $G \times_H P \rightarrow G \times_H X$ with isomorphic quotients. Thus, given an acyclic subcategory $I \subset \text{Res}_H(X)$ we obtain an acyclic subcategory $G \times_H I \subset \text{Res}_G(X)$. This may be used to deduce the “induction equivalence”

$$D_H(X) \xrightarrow{\sim} D_G(G \times_H X).$$

Composing this equivalence (or its inverse) with the functors of push-forward and pullback along the G -equivariant map $G \times_H X \rightarrow X$ yields the functors of **induction** and **restriction**:

$$\begin{array}{ccc} & \xrightarrow{\text{ind}_H^G} & \\ D_H^+(X) & & D_G^+(X) \\ & \xleftarrow{\text{res}_H^G} & \end{array}$$

They form an adjoint pair $(\text{res}_H^G, \text{ind}_H^G)$ and preserve the subcategories of bounded objects. A special case of this is when H is the trivial group in which case res_H^G is called the **forgetful functor** and denoted

$$\text{For} : D_G^+(X) \rightarrow D^+(X).$$

Note that ind_H^G does not commute with the forgetful functor in general.

Now suppose that $\phi : G \rightarrow H$ is a map of linear algebraic groups, X is a G -space, Y is an H -space, and $f : X \rightarrow Y$ is a morphism, equivariant with respect to ϕ . That is the following diagram commutes:

$$\begin{array}{ccc} G \times X & \longrightarrow & X \\ \phi \times f \downarrow & & \downarrow f \\ H \times Y & \longrightarrow & Y \end{array}$$

(where the horizontal morphisms are the action maps). In this case it is possible to define functors

$$\begin{array}{ccc} & f_* & \\ & \curvearrowright & \\ D_G^+(X) & & D_H^+(Y) \\ & \curvearrowleft & \\ & f^* & \end{array}$$

which in general do not preserve the full subcategories of bounded objects. (In fact it is because of these functors that we have been forced to consider the bounded below derived category from the outset.)

Let $I \subset \text{Res}_G X$ and $J \subset \text{Res}_H Y$ be acyclic subcategories. Pulling back the resolutions in J along f gives us a subcategory f^*J of the smooth maps to X . Moreover G acts on each variety in f^*J via the map $G \rightarrow H$. We denote by $I \times f^*J$ the category formed by taking fibre products of resolutions in I and morphisms in f^*J over X . Equipping each variety in $I \times f^*J$ with the diagonal action, $I \times f^*J$ becomes an acyclic subcategory of $\text{Res}_G(X)$. For all $P \in I$ and $Q \in J$ we have a map

$$\pi_{P,Q} : P \times_X f^*Q \rightarrow Q$$

commuting with the projection to $f : X \rightarrow Y$ and equivariant with respect to ϕ . We denote the induced map between the quotients by $\overline{\pi_{P,Q}}$.

We use the categories $I \times f^*J$ and J to define $D_G^+(X)$ and $D_H^+(Y)$ respectively. Pullback along $\overline{\pi_{P,Q}}$ defines a functor

$$f^* : D_H^+(Y) \rightarrow D_G^+(X).$$

We also obtain a functor

$$\begin{aligned} f_* : D_G^+(X) &\rightarrow D_H^+(Y) \\ (\mathcal{F}_{P,Q})_{P,Q} &\mapsto \left(\varprojlim_{P \in I} \pi_{P,Q,*} \mathcal{F}_{P,Q} \right)_{Q \in J}. \end{aligned}$$

Given maps $H \rightarrow G \rightarrow K$ of algebraic groups and equivariant maps $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ of G -, H - and K -spaces respectively it is

straightforward to see that the functors $g_* f_*$ and $(g \circ f)_*$ are naturally isomorphic.

A special case of f_* above is when H is the trivial group and Y is a point. In this case, for $\mathcal{F} = (\mathcal{F}_P)_{P \in I} \in D_G^+(X)$ we have

$$f_* \mathcal{F} = \varprojlim_{P \in I} \overline{\pi_P}^* \mathcal{F}_P$$

where $\overline{\pi_P} : \overline{P} \rightarrow pt$ is the map to a point. This is the equivariant hypercohomology of \mathcal{F} which we will sometimes denote by $\mathbb{H}_G^*(\mathcal{F})$.

6. Differential graded modules. The aim of this section is to give a description of a certain subcategory of the bounded G -equivariant derived category of mixed sheaves on a point in terms of differential graded modules.

Let $A = \oplus A_{ij}$ be a differential bigraded k -algebra with differential of degree $(1, 0)$. Recall that this means we have a bigraded unital k -algebra with differential $\partial = \oplus \partial_{ij}$ (where $\partial_{ij} : A_{i,j} \rightarrow A_{i+1,j}$) satisfying the Leibniz rule. We will consider the abelian category of differential bigraded right modules over A and its localization at quasi-isomorphisms, which we denote $\text{dgMod}A$. We will denote the full subcategory of finitely generated differential graded modules by $\text{dgMod}^f A$. The category $\text{dgMod}A$ (as well as $\text{dgMod}^f A$) is triangulated with shift functor given by

$$M[n]_{i,j} := M_{i+n,j}.$$

It also comes equipped with a **twist functor**, denoted $(m/2)$, and given by

$$M(m/2)_{i,j} := M_{i,j+m}$$

(the strange notation will look more natural later.) The cohomology of a differential bigraded algebra or module is the cohomology of the underlying chain complex of graded vector spaces.

Given a map $A \rightarrow A'$ of differential graded algebras determines functors of induction and restriction of scalars:

$$\begin{array}{ccc} & \overset{-\otimes_{AA}^L}{\curvearrowright} & \\ \text{dgMod}A & & \text{dgMod}A' \\ & \underset{\text{res}_A^{A'}}{\curvearrowleft} & \end{array}$$

For all of this we refer the reader to [BL94]. (They deal with the case of differential graded modules, but the case of bigraded modules is an obvious generalization.)

Let G_0 be a product of general linear groups over \mathbb{F}_q and G its extension to \mathbb{F} . Recall that we have fixed a square root of q in k and used it to define a square root of the Tate sheaf. Denote by $D_{G,1/2}^b(\text{pt})$ be the full triangulated subcategory of $D_{G,m}^b(\text{pt})$ generated by $\underline{k}_{\text{pt}}(1/2)^{\otimes m}$ for all $m \in \mathbb{Z}$.

Let $T_0 \subset G_0$ denote a split maximal torus and W the corresponding Weyl group. Denote by S the ring the symmetric algebra over k on the characters of T_0 . In formulas:

$$S := S(X(T_0) \otimes_{\mathbb{Z}} k).$$

(where $X(T_0)$ denotes the lattice of characters of T_0). We may give S a (pure) bigrading by specifying that elements of $X(T_0)$ live in degree $(2, 2)$. In fact we have isomorphisms of bigraded modules

$$H_T^*(\text{pt}) \cong S \quad \text{and} \quad H_T^*(\text{pt}) \cong S^W$$

where the bigradings on $H_T^*(\text{pt})$ and $H_T^*(\text{pt})$ are given by the cohomological and weight gradings.

Theorem 7. *With G as above one has an equivalence of triangulated categories*

$$\Gamma_G : D_{G,1/2}^b(\text{pt}) \xrightarrow{\sim} \text{dgMod}^f H_G^*(\text{pt})$$

commuting with the twist functors. Under this equivalence one has

$$\underline{k}_{\text{pt}} \cong H_G^*(\text{pt}) \quad \text{and} \quad \mathbb{H}_G^*(\mathcal{F}) \cong H^*(\Gamma_G(\mathcal{F})).$$

Moreover, if $G \subset K$ is an inclusion of products of general linear groups and $H_K^(\text{pt}) \rightarrow H_G^*(\text{pt})$ is the induced map then one has isomorphisms of functors:*

$$\begin{aligned} \Gamma_{H\text{res}_H^G}(-) &\cong \Gamma_G(-) \otimes_{H^*(G(\text{pt}))}^L H_K^*(\text{pt}), \\ \Gamma_{H\text{res}_H^G}(-) &\cong \text{res}_{H_G^*(\text{pt})}^{H_K^*(\text{pt})} \Gamma_G(-). \end{aligned}$$

Proof. By general arguments one has an equivalence between $D_{G,1/2}^b(\text{pt})$ and differential bigraded modules over the differential graded algebra of extensions of $\underline{k}_{\text{pt}}$. However the differential graded algebra in question is formal, being a limit of the cochains on smooth projective algebraic varieties. \square

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