

## The pilot-wave dynamics of walking droplets

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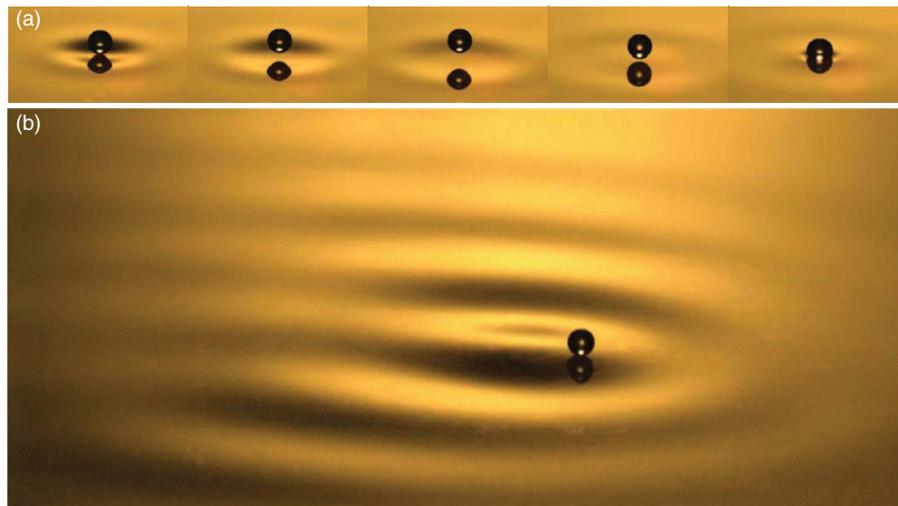


FIG. 1. (a) An oil droplet bouncing in place on a vibrating fluid bath. (b) At higher forcing amplitude, the droplet walks across the surface of the bath, propelled by its pilot-wave field (enhanced online). [URL: <http://dx.doi.org/10.1063/1.4820128.1>]

## The pilot-wave dynamics of walking droplets

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A millimetric droplet can be induced to bounce on the surface of a fluid bath by vibrating the bath near the droplet's resonant frequency (Figure 1(a)).<sup>1-3</sup> The localized field of Faraday waves excited by the bouncing droplet can cause it to propel itself laterally across the surface, moving in resonance with its guiding wave field (Figure 1(b)).<sup>4,5</sup> These walking droplets, or “walkers,” generally move in a straight line at constant speed; however, they can be diverted through interaction with boundaries or external forces. This hydrodynamic system represents a macroscopic realization of the pilot-wave theory of quantum dynamics proposed by Louis de Broglie, according to which microscopic particles are propelled through a resonant interaction with a wave field generated by the particle's internal vibration.<sup>6</sup> Coincidentally, it exhibits many behaviors once thought to be exclusive to the microscopic quantum realm, including single-particle diffraction,<sup>7</sup> tunneling,<sup>8</sup> quantized orbits,<sup>9</sup> and orbital-level splitting.<sup>10</sup>

We here investigate the dynamics and statistics of a walker confined to a circular corral. Tracking the motion of the walker (Figure 2(a)) indicates that its trajectory is irregular and its speed varies significantly along its path. Nevertheless, for a trajectory of sufficient length, a dependence of the velocity on position emerges (Figure 2(a), fourth panel). This dependence results in a coherent, wavelike statistical pattern for the walker's position (Figure 2(b)), with a wavelength prescribed by that of the droplet's guiding wave, the Faraday wavelength. The probability distribution is well described by the amplitude of the cavity's most unstable Faraday wave mode at the forcing frequency (Figure 2(c)). In the quantum corral experiments of Crommie *et al.*,<sup>11</sup> electrons were confined to a circular corral on the surface of a metal substrate. The electron's probability distribution function

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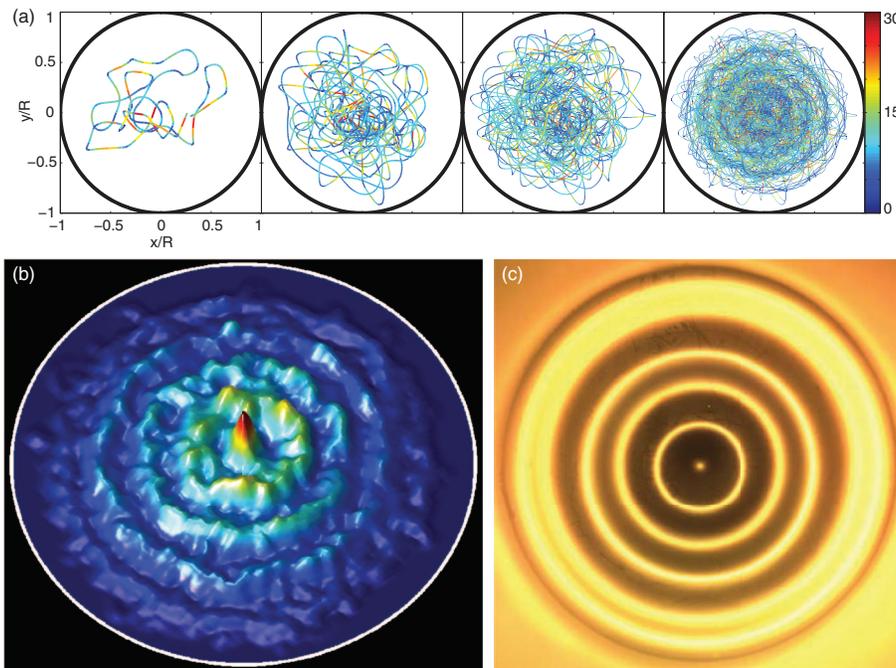


FIG. 2. (a) Trajectory of a walker confined to a circular cavity. Trajectories of increasing length are color-coded according to the droplet's local speed (mm/s). The duration of the trajectory presented in the fourth panel is approximately 21 min. (b) Probability distribution of the walker's position. (c) The Faraday wave mode of the cavity just above the Faraday threshold. The bright rings correspond to the extrema in the amplitude of the wave field.

took a similar circularly symmetric form, with a wavelength prescribed by the electrons' de Broglie wavelength. Whatever the case may be for quantum particles, the statistical description of our system is incomplete, and underlaid by a chaotic pilot-wave dynamics.

A more detailed account of our study is reported elsewhere.<sup>12</sup>

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