18.02A Problem Set 1 – Fall 2008 due Thursday 10/30/08, 12:45 in 2-106

18.02A Supplementary Notes: purchase in CopyTech, basement Bldg. 11.

Part I (20 points)

Do not hand in the exercises in parentheses, which are for more practice if you want or need it; hand in all the others.

Below, the notation 1A-2 means: Exercise 2 in Section 1A of the Exercises portion of the 18.02 Notes; it is solved in the Solutions section of the Notes. 17.3=section 17.3 of the textbook (Simmons 2nd ed.)

Read: 17.3, 18.1 Work: 1A-4a, 7bc, 8ab, 11 (1, 2, 6)

Read: 18.2 Work: 1B-1a, 3a, 5b, 11, 13

Read: 18.02 Notes D, pp. 1-3; 18.3 Work: 1C-2b, 3b, 4, 9; 1D-1b, 2, 5, 6

Read: Notes M.1, M.2 Work: 1F-5b, 8a; 1G-3, 4, 5

Lecture 23. Tues. Oct. 28 Theorems about square systems; Equations of planes
Read: Notes M.3, M.4 Work: 1H-3abc, 7. Read: pp. 648, 9 Work: 1E-1cd, 2

Read 18.4, 17.1

Part II (30 points)

Directions. Try each problem alone for 15 minutes. If you subsequently collaborate, solutions must be written up independently. It is illegal to consult problem sets from previous semesters. With each problem is the day it can be done.

Problem 1. (Tues. 4 pts) A man travelling east at speed U finds that the wind seems to blow directly from the north. On doubling his speed, he finds it appears to come from the northeast. Find the velocity (speed and direction) of the wind.

Problem 2. (Tues. 4 pts) Show that the lines joining the midpoints of the sides of a quadrilateral form a parallelogram.

Problem 3. (Thurs. 2 pts) Prove the trigonometric formula

\[ \sin(\theta_2 - \theta_1) = \sin \theta_2 \cos \theta_1 - \sin \theta_1 \cos \theta_2 \]

by interpreting both sides as the area of a parallelogram formed by the unit vectors passing through the origin and making angles \( \theta_1 \) and \( \theta_2 \) with the \( x \)-axis.

Problem 4. (Thurs. 4 pts: 2+2) The Biot-Savart law, which you will study in 8.02, says that the magnetic field \( \mathbf{B} \) at a point with position vector \( \mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = (x, y, z) \) induced by a current
of magnitude \( J \) passing through the origin \((0, 0, 0)\) in the \( \hat{k} \)-direction is given by

\[
B = \left( \frac{\mu_0}{4\pi} \right) J \frac{\hat{k} \times r}{r^3},
\]

where \( \mu_0 \) is a physical constant and \( r = |r| \).

a) Find the formula for the magnetic field \( B \) at a general point \((x, y, z)\) and evaluate it at the point \((1, 2, 3)\).

b) For which points \((x, y, z)\) at unit distance from the origin is the magnitude of the magnetic field, \( |B| \), largest?

**Problem 5.** (Thurs. 4 pts: 2, 2) Let \( A = 2i - j + 2k \) and \( B = 2i + 2j - k \)

a) Show that \( A \) and \( B \) are perpendicular.

b) Express \( C = 3i - j + 2k \) in terms of the primed unit vectors.

One way to do this is to solve backwards for \( i, j, k \) in terms of \( i', j', k' \) and then substitute. An easier way however is to observe that we are looking for the components of \( C \) in the directions of the primed unit vectors, and use the fact that \( V \cdot u \) is the component of \( V \) in the \( u \)-direction. Use the latter method.

**Problem 6.** (Fri. 4 pts: 2,2) Do part b) of the previous problem as follows.

a) Using a suitable 3 by 3 matrix \( M \), write symbolically

\[
M[i \ j \ k]^T = [i' \ j' \ k']^T,
\]

then calculate \( M^{-1} \).

(To make the calculations look better, factor out of \( M \) the common denominator of its entries, writing \( M = cN \), where \( N \) has integer entries. Then find \( N^{-1} \) and convert it to \( M^{-1} \). \( T \) denotes the transpose.)

b) To express \( C \) in terms of the primed unit vectors, do the substitution nicely by writing symbolically \( C = V[i \ j \ k]^T \) for some suitable row vector \( V \), and then substitute for the column vector and use matrix multiplication.

**Problem 7.** (Fri. 4 pts: 1+1+1+1) An \( n \) by \( n \) matrix \( A \) is **orthogonal** if \( A(A^T) = (A^T)A = 1 \). Consider the rotation matrix:

\[
A = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

a) Show that \( A_\theta \) is **orthogonal**.

b) Find \( u = A_\theta i \) and \( v = A_\theta j \) and draw a picture of \( u \) and \( v \) for \( \theta = \pi/4 \).

c) Use the fact that \( A_{\theta_1} A_{\theta_2} = A_{\theta_1 + \theta_2} \) to deduce the addition formulas for sine and cosine.

d) Using the concept of rotation, rationalize why \( A_\theta^{-1} = A_{-\theta} \). Compute \( A_\theta^{-1} \) and compare it with \( A_{-\theta} \) to show that this is indeed the case (use the properties of sine and cosine).

**Problem 8.** (Tues. 4 pts) For what value of \( \lambda \) do the four points \((0, -1, -1), (3, 9, 4), (-4, 4, 4) \) and \((4, 5, \lambda)\) lie in a plane?