Problem 1 (25 pts: 5, 10, 10) The rectangular box has edges of lengths 1, 2, 3 lying along the coordinate axes, as shown. Using head-to-tail addition, \( RQ = -2i + k \).

a) Express similarly \( PQ \) and \( PR \) in terms of \( i, j, \) and \( k \).

b) Find the cosine of the angle \( \theta = RPQ \).

c) Find a perpendicular vector to the plane through \( P, Q, \) and \( R \), and find the area of triangle \( PQR \).

Problem 2. (25 pts: 10, 10, 5)

\[
\begin{bmatrix}
1 & 0 & 1 \\
2 & 1 & 2 \\
1 & 2 & 3
\end{bmatrix}
\]

a) If \( A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \), fill in the missing four entries in:

\[
A^{-1} = \begin{bmatrix} -1/2 & 1 & -1/2 \\ 3/2 \\ -2 \end{bmatrix}
\]

b) Solve \( x_1 + x_3 = 2 \) if \( c = 3 \), using part (a).

\[
\begin{align*}
2x_1 + x_2 + 2x_3 &= 1 \\
x_1 + 2x_2 + cx_3 &= 2
\end{align*}
\]

Then check the value of \( x_1 \) by using Cramer's rule. (Show work.)

c) For one value of \( c \) the system in (b) has no solution. Find this value.

Problem 3. (25 pts: 5, 10, 10)

A moving point \( P \) has coordinates \( x = (t - 1)^2, \ y = t^2, \ z = 2t - 1 \).

a) Where (i.e., at what point) does it pass through the \( yz \)-plane?

b) At time \( t = 0 \), find its speed and the unit tangent vector to the motion.

c) Find a constant vector perpendicular to the position vector \( \mathbf{R} = \mathbf{OP} \). What does this tell you geometrically about the motion?

Problem 4. (10 pts.) Scotch tape is being peeled off a roll of radius \( a \), starting at the point \( A \). The end \( P(x,y) \) is always pulled vertically upwards. Use vector methods to write parametric equations for \( x \) and \( y \) terms of \( \theta \), for \( 0 \leq \theta \leq \pi \).

(See fig. below)

Problem 5. (10 pts.) Using the standard operations on vectors, prove that if the diagonals of a parallelogram are perpendicular, its four sides have equal length.

Problem 6. (5 pts). A point moves with constant speed. Prove its velocity vector \( \mathbf{v} \) and its acceleration vector \( \mathbf{a} \) are always perpendicular. (Consider \( \mathbf{v} \cdot \mathbf{v} \).)