HOMEWORK 8 FOR 18.747, SPRING 2013
DUE FRIDAY, APRIL 12 BY 3PM.

In all problems G is a connected linear algebraic group over a field k.

(1) Let $B \subset G$ a Borel subgroup and set $\mathcal{B} = G/B$.
For $u \in G$ let $\mathcal{B}_u = \{x \in \mathcal{B} \mid u(x) = x\}$, this is a closed subvariety in $\mathcal{B}_u$.
   (a) Let $G = GL_n$ ($n \geq 3$), let u be a unipotent element with $\dim Ker(u-1) = 2$, $\dim Ker(u-1)^2 = 3$. Show that $\mathcal{B}_u$ is a union of $n-1$ irreducible components, each isomorphic to $\mathbb{P}^1$. Describe the pattern of intersection of these components.
   (b) (Optional) Let $G = Sp_4$. Find a unipotent $u \in G$ such that $\dim(\mathcal{B}_u) = 1$. Show that such elements form one conjugacy class. For such an element u show that G_u has three irreducible components, L_0, L_1, L_2, with $L_i \cong \mathbb{P}^1$ and such that L_0 intersects L_1, L_2 at one point but $L_1 \cap L_2 = \emptyset$. Find $s \in G$ commuting with u such that s permutes L_1 with L_2.

(2) Let $g = su$ be an element in G with Jordan decomposition. Check that u lies in the connected component of identity of the centralizer of s.

(3) Let C be a maximal nilpotent closed subgroup in G such that C is the connected component of identity in its normalizer. Then C is a Cartan subgroup.