

HOMEWORK 11 FOR 18.747, SPRING 2013
DUE FRIDAY, MAY 3 BY 3PM.

- (1) Check that the centralizer of a semi-simple element in a reductive group is reductive.
- (2) Let G be a semi-simple connected algebraic group of rank one over a field k of characteristic different from two. Let V be the space of functions on G satisfying $f(gb) = \chi(b)f(g)$ for $b \in B$. Here B is a Borel subgroup and $\chi : B \rightarrow \mathbb{G}_m$ is the composition of the projection $B \rightarrow B/U$ and an isomorphism $B/U \cong \mathbb{G}_m$. Assume that χ is chosen so that $V \neq 0$.

Show that the space V carries a nontrivial G -invariant bilinear form, and G maps isomorphically to the stabilizer of that form in $GL(V)$.