HOMEWORK 10 FOR 18.747, SPRING 2013
DUE FRIDAY, APRIL 26 BY 3PM.

As before G is a connected linear algebraic group over an algebraically closed field and $T \subset G$ is a maximal torus and W is the Weyl group.

(1) Show that an element of the Weyl group is a product of at most r reflection, where r is the rank of G.
(2) Check that if $w \in W$ fixes an element x of $X^*(T) \otimes_{\mathbb{Z}} \mathbb{R}$, then it equals a product of reflections s_α, such that each s_α fixes x.
(3) Let $G = SL_n$ over k of characteristic zero.
   (a) Check that the homomorphism $N(T) \to N(T)/T = W \cong S_n$ admits a one-sided inverse iff n is odd.
   [Hint: for n even look at a representative s of the long cycle and compute s^n].
   (b) Recall that S_n is generated by elementary transpositions $\sigma_i = (i, i+1)$ subject to relations $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$, $\sigma_i \sigma_j = \sigma_j \sigma_i$ for $|i-j| > 1$, $\sigma_i^2 = 1$. [You are not asked to prove this].
   Show that $N(T)$ contains a subgroup $\tilde{S}_n$ generated by elements $\tilde{\sigma}_i$ which satisfy the relations $\tilde{\sigma}_i \tilde{\sigma}_{i+1} \tilde{\sigma}_i = \tilde{\sigma}_{i+1} \tilde{\sigma}_i \tilde{\sigma}_{i+1}$, $\tilde{\sigma} \tilde{\sigma}_j = \tilde{\sigma}_j \tilde{\sigma}_i$ for $|i-j| > 1$, $\tilde{\sigma}_i^4 = 1$.