HOMEWORK 1 FOR 18.747, SPRIN 2013
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(1) Let $X \subset \mathbb{A}^2$ be the union of the two coordinate axes. Show that X can not be endowed with the structure of an algebraic group.

(2) Let $\phi : G \to H$ be a homomorphism of linear algebraic groups and $\phi^* : O(H) \to O(G)$ the corresponding homomorphism of coordinate rings. Let $I_e \subset O(H)$ be the ideal of functions vanishing at the unit $e_H \in H$. Set $I = \phi^*(I_e)O(G)$. Check that $A = O(G)/I$ defines an affine group scheme; we will call it the *scheme-theoretic kernel* of ϕ.

(3) An affine group scheme is finite if the corresponding coordinate algebra A is finite dimensional. It is called commutative if the image of $\delta : A \to A \otimes A$ is contained in the space of tensor invariant under the involution $f \otimes g \mapsto g \otimes f$. Check that if A corresponds to a commutative finite group scheme G, then the dual vector space A^* also has a structure of a commutative Hopf algebra, so that it corresponds to a finite group scheme, which we denote by G'.

(4) Assume that k has characteristic $p > 0$. Let G the scheme-theoretic kernel of the map $\mathbb{G}_m \to \mathbb{G}_m$, $t \mapsto t^p$. Show that G' is a finite commutative group scheme and describe G'.