

### List of topics for the final paper for 18.745, due 12/10

The paper should include some (though maybe not all) proves. A reasonable volume for the paper is about 5-10 pages, talk to me if there are questions. If you prefer to opt for a take home final, let me know at least 3 weeks in advance. The list may be modified/extended during the semester. You are also welcome to suggest a topic of your choice.

1. An explicit model for one (or more) of the exceptional Lie algebras, see Encyclopedia of mathematics, vol. 41, ch. 5 and references therein.
2. Spinor representations of real and complex orthogonal Lie algebras, see Fulton, Harris "Representation theory: a first course", section 20.
3. Restricted Lie algebras in characteristic  $p$ , representations of  $sl(2)$  over an algebraically closed field of positive characteristic. (Jacobson's book)
4. Campbell-Hausdorff formula (Serre's book).
5. MacKay correspondence between finite subgroups in  $SL(2)$  and simple Lie algebras, isomorphism between the subregular slice and the quotient of the plane by the finite group: formulation, computation for  $SL(n)$  and at least one other example. [Requires some algebraic geometry and finite group representation theory].
6. Coxeter transformation and exponents (Bourbaki ch V, §6 + some of the exercises)
7. Description of the ring of invariant polynomials on a semi-simple Lie algebra (a part of: Bourbaki, ch VII, §§8.1–8.3).
8. Lie algebra cohomology, description of cohomology of a simple Lie algebra. Cohomology with coefficients in a module, application to Levi Theorem.
9. The center of the enveloping algebra: the Harish-Chandra isomorphism (Bourbaki, ch VII, §§8.5).
10. Ado Theorem.
11. Hopf algebras, deformation of enveloping algebras to quantum enveloping (parts of: Etingof, Schiffmann, "Lectures on quantum groups" chapters 8,9, or Kassel "Quantum groups", chapter 3,5).

12. Generators and relations for quantum  $U_q(2)$ , representations for various values of  $q$  (Jantzen, "Lectures on quantum groups").