MATH 18.726, SPRING 2015 - PROBLEM SET # 8

(1) Harthshorne III.10.1.
(2) Harthshorne III.10.3.
(3) Harthshorne III.10.4
(4) (Eisenbud, problem 18.10) F, G are homogeneous polynomials in three variables over an algebraically closed field. Assume that the corresponding curves in $\mathbb{P}^3$ meet transversely at a finite set of points. Show that the ideal of homogeneous polynomials vanishing at those points is generated by F, G.

    [Hint: check that $k[x, y, z]/(F, G)$ has no submodules supported at zero].
(5) (Eisenbud, problem 18.16) The homogeneous coordinate ring of a projective variety $X \subset \mathbb{P}^r$ of pure dimension d is Cohen-Macaulay iff $H^1(\mathbb{P}^r, \mathcal{I}(n) = 0$ and $H^i(\mathcal{O}_X(n) = 0$ for $0 < i < d$ and all n. Here $\mathcal{I}$ is the ideal sheaf of X.