

HOMEWORK 2 FOR 18.726, SPRING 2015
DUE FRIDAY, FEBRUARY 20 BY 3PM.

- (1) # 3.5
- (2) # 3.13(f).
- (3) # 3.8
- (4) Let M be the submonoid in \mathbb{Z}^2 generated by $(0, 1)$, $(2, 1)$, $(3, 1)$. Let k be a field. Let $X = \text{Proj}(k[M])$ and Y be its normalization. Show that the map $Y \rightarrow X$ is a homeomorphism but is not an isomorphism.
- (5) (Optional bonus problem). Let A be a finitely generated commutative ring over a field k . Let $d = \dim(\text{Proj}(A))$. Show that for some constants $c_1, c_2 > 0$ we have $c_1 n^{d+1} \leq \dim(A^{\leq n}) \leq c_2 n^{d+1}$ for all $n > 0$.