**HOMEWORK 6 FOR 18.706, FALL 2018**
**DUE MONDAY, DECEMBER 10.**

(1) Let $R$ be a prime PI-algebra satisfying an identity of degree $d$. Show that the left (or right) uniform rank of $R$ is less than $d$.
(2) Prove that if $s \in R$ is regular and ad nilpotent then $GK \dim R[s^{-1}] = GK \dim(R)$.
(3) (GK dimension does not behave well on short exact sequences)
Show that the following provides an example of a PI algebra $R$, an $R$-module $M$ with a submodule $N$, s.t. $GK \dim(N) = GK \dim(M/N) = 1$, $GK \dim(M) = 2$.
Set $R = \mathbb{C}\langle x, y\rangle/yx = 0$, let $M$ have two generators $\alpha, \beta$ subject to relations: $x^{n+1}y^n\alpha = 0$ and $xy^n\beta = 0$ unless $n$ is a square $m^2$ in which case $xy^n\beta = xy^m\alpha$. Let $N = R\beta$.
Check that $R$ satisfies the identity $[a, b]^2 = 0$, thus it is PI.
(4) Show that the enveloping algebra $U(sl(2, k))$ is a PI algebra iff the field $k$ has positive characteristic.
(5) Let $G$ be the group of transformations of the line generated by $x \mapsto x + 1$ and $x \mapsto 2x$. Show that the group algebra of $G$ has exponential growth.