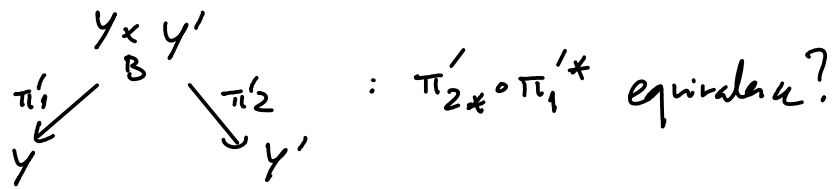




Guess #1:  $\pi_2 \times \pi_1^*$  is an equivalence?

This is not true (Namikawa, Kawamata)

Guess #2: use, instead of  $W$ ,



Namikawa-Kawamata: || for  $k=1$  this is an equivalence

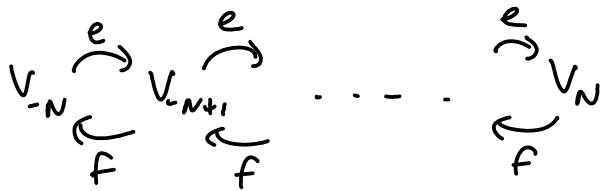
but... Namikawa: this fails for  $T^*Gr(2,4)$

→ dealt with by Kawamata.

Recall: rep of  $SL(2)$  break up into weight spaces

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



$\exists$  natural isom.  $V_{-\lambda} \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} V_{\lambda}$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \frac{e^{\lambda}}{\lambda!} - \frac{e^{\lambda+1}f}{(\lambda+1)!1!} + \frac{e^{\lambda+2}f^2}{(\lambda+2)!2!} + \dots$$

Categorify this:

Let  $D_{\lambda} := D(T^*Gr(l, N))$  where  $N$  fixed,  $N-2l = \lambda$

Then want functors  $D(T^*Gr(l+1, N)) \begin{array}{c} \xrightarrow{E} \\ \xleftarrow{F} \end{array} D(T^*Gr(l, N)) \begin{array}{c} \xrightarrow{E} \\ \xleftarrow{F} \end{array} \dots$

$E$  is given by

$$W = \{ (x, v, v') : 0 \subset v^l \subset v'^{l+1} \subset \mathbb{C}^N \}$$

forget  $v$  ↙

$T^*Gr(l+1, N)$



↘ forget  $v'$

$T^*Gr(l, N)$

Similarly for  $F$ .

Relation  $ef - fe = h$  categorifies to:  $E \circ F = F \circ E \oplus \text{Id}^{\oplus \lambda}$   
 (for  $\lambda > 0$ )

More generally,  $E^{(m)} : \mathcal{D}(T^* \text{Gr}(l+m, N)) \rightarrow \mathcal{D}(T^* \text{Gr}(l, N))$   
 via  $W = \{(x, y, v) \mid 0 \subset V^l \subset V^{l+m} \subset \mathbb{C}^N\}$   
 categorifies  $e^{(m)} = \frac{e^m}{m!}$

We have relation  $E^{(m)} \circ E = E^{(m+1)} \oplus_{\mathbb{C}} H^*(\mathbb{P}^m)$   
 categorifies  $e^{(m)} \cdot e = \frac{e^{m+1}}{m!} = (m+1) e^{(m+1)}$

Now we can build the terms  $E^{(\lambda)}, E^{(\lambda+1)} \circ F^{(1)}, E^{(\lambda+2)} \circ F^{(2)}, \dots$   
 needed to categorify  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = e^{(\lambda)} + e^{(\lambda+1)} f + e^{(\lambda+2)} f^{(2)} + \dots$

Construct a complex  $(*)$   $E^{(\lambda)} \rightarrow E^{(\lambda+1)} \circ F \rightarrow E^{(\lambda+2)} \circ F^{(2)} \rightarrow \dots$   
 $\rightarrow$  each of these functors has a kernel which is a sheaf on the appropriate  $W$ , so we construct the kernel by constructing natural maps between the relevant kernels, which come from the fact that  $E, F$  are mutually adjoints.

Now, consider  $\| \text{Cone}(E^{(\lambda)} \rightarrow E^{(\lambda+1)} \circ F \rightarrow E^{(\lambda+2)} \circ F^{(2)} \rightarrow \dots) =: T(\lambda).$

This  $T(\lambda)$  gives the desired equivalence for the stratified flop.

Thm (Cant's - Kamnitzer - Licata):

$\|$  1. If you have a strong categorified  $\mathfrak{sl}_2$ -action then you can construct the complex  $(*)$  and  $\text{Cone}(*) : \mathcal{D}(\lambda) \rightarrow \mathcal{D}(-\lambda)$

Thm (cont'd)

2.  $E^{(i)}, F^{(j)}$  as above form a strong categorified action,  
and yield  $\text{Core}(*) : \mathcal{D}(T^*Gr(k, N)) \cong \mathcal{D}(T^*Gr(N-k, N))$

Point: strong  $sl_2$ -action has not only  $E \circ F \simeq F \circ E \oplus Id^\uparrow$ ,  
but also extra natural transformations

$$X: E[-1] \rightarrow E[1]$$

$$T: EE[1] \rightarrow EE[-1]$$

satisfying (1)  $T^2 = 0$

(2) on  $\underbrace{EEE}_{T_1 \text{ acts here}}$ ,  $T_1 T_2 T_1 = T_2 T_1 T_2$   
 $T_2$  acts here

(3) on  $\begin{matrix} EE \\ \uparrow \quad \uparrow \\ x_1 \text{ acts here} \quad x_2 \text{ acts here} \end{matrix}$ ,  $TX_1 - X_2T = Id.$