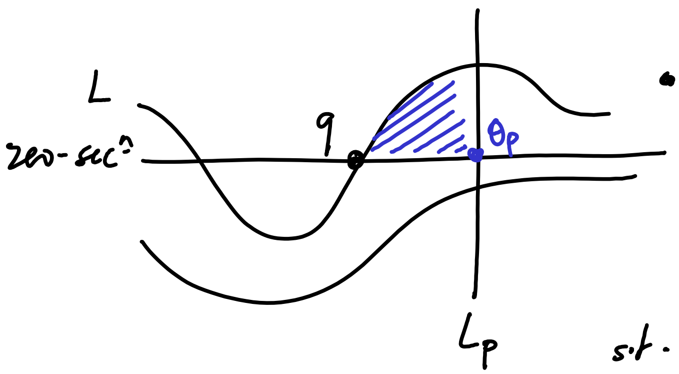
K. Fukaya – 22/1/09 – More about SYZ & HMS

(see: – Floer homology for families
* Mirror symm. for abelian varieties, J Alg-Geom.

1) Case L multisection in semiflat SYZ fibration

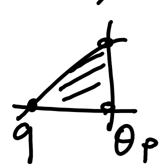


• $\mathcal{E} = \left(CF^*(L, L_p)\right)$ is a collection of vector bundles on the SYZ mirror

• carries natural holomorphic structure s.t. sections defined below are holomorphic
($\Leftrightarrow$ define holom. str. by SYZ-transform recipe: $\nabla = d + \sum f_i\, dy_i$ where $L = \text{graph}(\sum f_i dx_i)$)

• $q \in L \cap 0$-section of Maslov index 0
$\Rightarrow$ section s_q, s.t. $s_q(p) = m_2(q, \theta_p) \in \underbrace{CF^*(L, L_p)}_{\text{fiber at } p \checkmark}$

• $q \in L \cap 0$-section of Maslov index k
$\Rightarrow$ k-form $s_q \in \Omega^{0,k}(\mathcal{E})$
$\rightarrow$ virt. dim. for discs



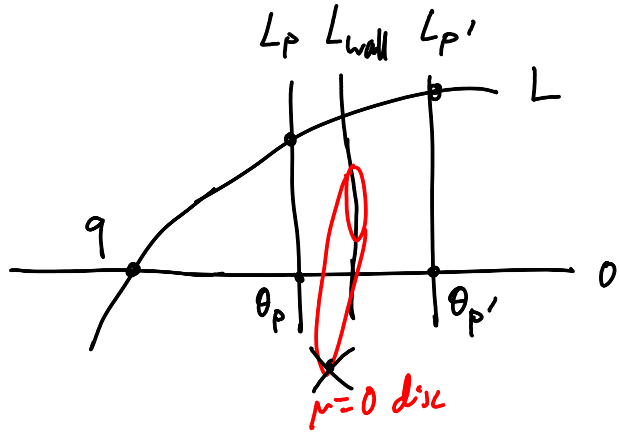
is $-k$

so $\exists$ discs only if $p \in$ codim. k submfld of the base.
Dual current $\in \Omega^{0,k}$, then m_2 gives k-form in $\mathcal{E}$
(current supported along codim. k subvar!)

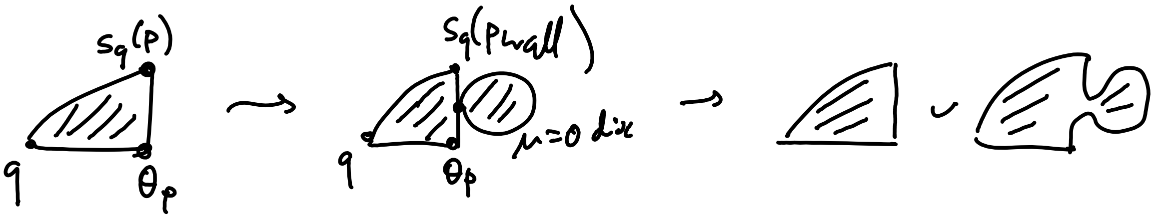
(current should smooth to form by picking perturbation and $\int_{\text{pert. space}}$)

2) If ambient space has sing. fibers

$\Rightarrow$ mirror has instanton corrections...

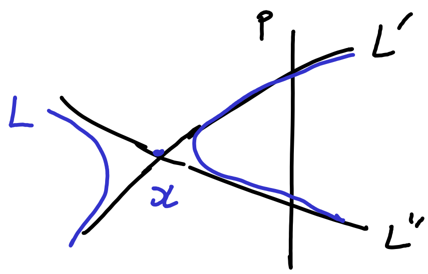


e.g.



s_q is going to be holomorphic on corrected mirror

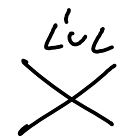
3) If L has caustics... view it as an extension b/w multisections



gives a complex

$$0 \to \mathcal{E}(L') \to \mathcal{E}(L) \to \mathcal{E}(L'') \to 0$$

$$\mathcal{E}(L') \oplus \mathcal{E}(L'') \text{ would be}$$



now: complex $\begin{pmatrix} \bar\partial & ▨ \\ 0 & \bar\partial \end{pmatrix}$ where $▨ \in \mathrm{Hom}(\mathcal{E}(L''), \mathcal{E}(L'))$

is given by $m_2(x, \cdot) : CF(L_p, L'') \to CF(L_p, L')$