Discussion session (19/1/09) (moderator: P. Seidel)

<u>Q^n</u> (Soibelman) – What's HMS for the resolved conifold?

<u>Ans:</u> (Seidel + ε)

$$X = \mathrm{Tot}\big(\mathcal{O}(-1)\oplus\mathcal{O}(-1) \longrightarrow \mathbb{P}^1\big) \qquad \left(\begin{array}{l}\text{Complex: rigid}\\ \text{sympl: area param.}\end{array}\right)$$

$$\updownarrow ?$$

$$X^\vee = \{x_0^2 + x_1^2 + x_2^2 = z + az^{-1}\} \subset \mathbb{C}^3 \times \mathbb{C}^* \qquad \left(\begin{array}{l}\text{sympl: rigid}\\ \text{Complex: smoothing param. } a\end{array}\right)$$

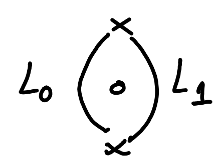
- $D^b_{\mathbb{P}^1}(X) \xleftarrow{\;F\;} D^b(\mathbb{P}^1)$

Image of F generates $D^b_{\mathbb{P}^1}(X)$, hence generators $\mathcal{O}_{\mathbb{P}^1}$, $\mathcal{O}_{\mathbb{P}^1}(1)$

$$\begin{tikzcd} \mathcal{O}_{\mathbb{P}^1} \arrow[r, bend right, "Ext^0 = \mathbb{C}^2"'] & \mathcal{O}_{\mathbb{P}^1}(1) \arrow[l, bend right, "Ext^3 = \mathbb{C}^2"'] \end{tikzcd}$$

while on the mirror: $\qquad X^\vee \xrightarrow{\;\pi\;} \mathbb{C}^*$
(take $a=1$) $\qquad (x_0 \ldots x_2, z) \longmapsto z$

conic bundle
sing. fibers at $z = \pm i$

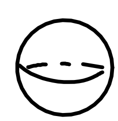


matching paths
$\to L_0, L_1 \subset X^\vee$
Lagrangian spheres
$\hom(L_0, L_1) = \mathbb{C}^2$ in degree 0

$L_0, L_1 \subset X^\vee$ mirror to $\mathcal{O}_{\mathbb{P}^1}, \mathcal{O}_{\mathbb{P}^1}(1)$

- Other side of the mirror:

$\mathcal{O}$ on $X^\vee$ $\longleftrightarrow$ noncompact Lag. $S^1 \times \mathbb{R}^2 \subset X$

projects to equator  $S^1 \subset \mathbb{P}^1$

& $\mathbb{R}^2 \subset \mathbb{C}^2$ for each fiber above equator

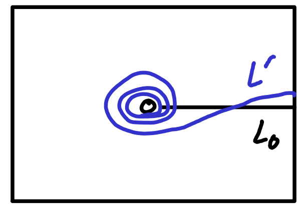
- **Q:** Mirror to $\mathcal{O} \oplus \mathcal{O}(-2) \longrightarrow \mathbb{P}^1$?

  ans: $\mathbb{C}$ factor splits off $\Rightarrow$ (mirror to $\mathbb{C}$) $\times$ (mirror to $\begin{matrix}\mathcal{O}(-2)\\ \downarrow \\ \mathbb{P}^1\end{matrix}$)

  - mirror to $\mathcal{O}(-2) \to \mathbb{P}^1$ is $\{x_0^2 + x_1^2 = z + a z^{-1}\}$
  - mirror to $\mathbb{C}$ is $\mathbb{C}^*$ with $\begin{cases} - \text{ small pert. at } \infty \\ - \text{ wrapping at } 0 \end{cases}$

  e.g. $\mathcal{O} \longleftrightarrow L_0 = \mathbb{R}_+$,

  (Gabriel KERR)

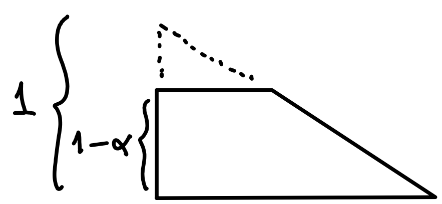


$$\mathrm{Hom}(L_0, L_0) := HF(L_0, L') = \mathbb{C}[t] \ \checkmark$$

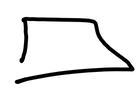
Q (Seidel): what's the mirror to $\mathbb{F}_n$?

A: (Fukaya)

Mirror to $\mathbb{F}_n$ is
$$W = y_1 T^{u_1} + y_2 T^{u_2} + y_1^{-1} y_2^{-n} T^{n - u_1 - n u_2} + y_2^{-1} T^{1 - \alpha - u_2}$$



Allow $y_i \notin \Lambda^+$ ($\Leftrightarrow$ allow $\mathbb{C}^*$ in complex setup)

Solve $\frac{\partial W}{\partial y_1} = 0 = \frac{\partial W}{\partial y_2}$ ... 4 crit pts inside 
others outside.

- - - - - - - - - - -

A: (Auroux)

Corrected Mirror to $\mathbb{F}_3$: $W = \underbrace{x + y + \frac{B}{y} + \frac{AB^2}{xy^3}}_{\text{usual mirror}} + \underbrace{\frac{2AB}{y^2} + \frac{A^2B^2}{xy^4}}_{\text{contribution from } (\mu=4 \text{ disc}) \cup (\text{exc. curve})}$

has only 4 crit pts