Let $Z_n$ be the group of unipotent matrices of order $n+1$ and let $z = (z_{ij})$, $0 \leq i \leq j \leq n$, with $z_{ii} = 1$, be matrices from $Z_n$. The $n$-dimensional torus consisting of diagonal matrices $t = \text{diag}(t_0, t_1, \cdots, t_n)$, $t_0 t_1 \cdots t_n = 1$, acts on $Z_n$ by conjugation: $z \equiv \{z_{ij}\} \rightarrow \{z_{ij} t_i t_j^{-1}\}$. The following system of differential equations is related to this action:

$$
- \sum_{i=0}^{j-1} z_{ij} \frac{\partial f}{\partial z_{ij}} + \sum_{k=j+1}^{n} z_{jk} \frac{\partial f}{\partial z_{ij}} = \alpha_j f, \quad j = 0, 1, 2, \cdots, n,
$$

$$
\frac{\partial f}{\partial z_{ik}} = \frac{\partial^2 f}{\partial z_{ij} \partial z_{jk}},
$$

$0 \leq i < j < k \leq n$, where $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_n)$ and $\sum \alpha_i = 0$. It is called the hypergeometric system on the group of unipotent matrices. Solutions of this system are called hypergeometric functions on this group. This hypergeometric system gives a holonomic $D$-module. The authors find the number of independent solutions of this system at a generic point. This number is equal to the Catalan number. An explicit basis of $\Gamma$-series in the solution space of the system is constructed. The restriction of this system to certain strata is considered. Several combinatorial constructions with trees, polyhedra and triangulations, related to this system, are introduced and studied.

{For the entire collection see MR 97g:00016.}

A. Klimyk (Zbl 876.33011)