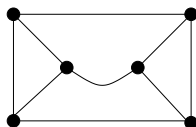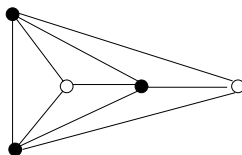# Exam 3

December 08, 2005

You have 1 hour 15 min to solve the following problems. The problems worth 10 points each. You can use your notes, books, calculators, etc. Show your reasoning.

1. Calculate the number of spanning trees in the "envelope" graph:
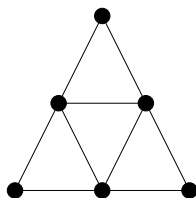

2. Find the total resistance between two white vertices in the graph:

   Assume that resistances of all edges in the graph are 1.

3. Calculate the chromatic number, the chromatic polynomial, and the number of acyclic orientations for the graph:

4. (a) For two positive integers $m < n$, find the number $f(m,n)$ of ways to extend a given tree T on the vertices $1, \ldots, m$ to a tree on the vertices $1, \ldots, n$, by adding edges. For example, $f(2,3) = 2$ and $f(2,4) = 8$.

   (b) More generally, suppose that F is a fixed forest on n vertices with k connected components of sizes $n_1, \ldots, n_k$. (We have $n = n_1 + \cdots + n_k$.) Find the number of trees on the same vertex set that contain the forest F.