Problem 1. Find the minimal number of adjacent transpositions needed to transform the permutation 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 into the permutation 6, 7, 8, 9, 10, 1, 2, 3, 4, 5.

Problem 2. Fix two nonnegative integers $m$ and $n$. Find a closed expression for the sum of binomial coefficients
$$\sum_{a=0}^{m} \sum_{b=0}^{n} \binom{a+b}{a}.$$

Problem 3. For an integer $n \geq 3$, prove the following identity for $q$-binomial coefficients:
$$\left[ \begin{array}{c} n + 3 \\ n \end{array} \right]_q = \sum_{k=0}^{3} q^{k^2} \left[ \begin{array}{c} 3 \\ k \end{array} \right]_q \left[ \begin{array}{c} n \\ k \end{array} \right]_q.$$

Problem 4. Prove that the number of compositions of $n$ with all parts greater than or equal to 2 equals the Fibonacci number $F_{n-1}$.

Problem 5. Prove that the number of compositions of $n$ with all odd parts equals the Fibonacci number $F_n$.

Problem 6. Fix two integers $n \geq 0$ and $k \geq 2$. Show that the following 3 numbers are equal:
(a) The number of compositions of $n$ with parts equal to 1 or $k$.
(b) The number of compositions of $n + k$ with all parts greater than or equal to $k$.
(c) The number of compositions of $n + 1$ with all parts congruent to 1 modulo $k$.

Problem 7. Use the formula $x^n = \sum_{k=0}^{n} S(n, k) (x)_k$ to find explicit expressions for the Stirling numbers of the second kind $S(n, 1)$, $S(n, 2)$, $S(n, 3)$ and $S(n, 4)$ as linear combinations of $1^n, 2^n, 3^n,$ and $4^n$.

Problem 8. Let $p(n)$ denote the number of integer partitions of $n$. Show that $p(n) - p(n - 1)$ equals the number of integer partitions of $n$ with all parts greater than or equal to 2.

Problem 9. Fix two integers $n \geq 0$ and $k \geq 1$. Show that the following 3 numbers are equal:
(a) The number of integer partitions of $n$ with at most $k$ parts.
(b) The number of integer partitions of $n + k$ with exactly $k$ parts.
(c) The number of integer partitions of $n$ with all parts less than or equal to $k$. 

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Problem 10. Show that the number of integer partitions of $n$ with odd parts equals the number of integer partitions of $n$ with all distinct parts.