18.01A Problem Set 4A due Thurs. Oct. 15, 2009 12:45 2-106

Compared with the earlier Problem Set 4A, Parts I and II now both contain material relevant to the Wed. 10/15 recitation; also the earlier Part II, Problem 1b has been deleted.

Part I (15 pts.)

Lecture 13. Thurs. Oct. 8 Infinite series: ratio test, absolute convergence.

Geometric series; convergence and divergence. Read: pp. 439-440

Comparison tests: review simple and asymptotic comparison ("limit comparison" in the book): Read pp. 451-2 Ratio test: Read pp. 461-3 top (through Ex. 3).

Absolute and conditional convergence; alternating series test.

Read: 13.8 (you can skip the proof on p. 466)

Work: 7B-4abdf; 7B-5eghi

Lecture 14. Fri. Oct. 9 Power series: radius of convergence, diff'n and integ'n.

Radius of convergence: Read 483-5; p. 486 bottom (for def'n of R). (Don't use formula (7); use the ratio test on the power series itself, as in lecture and the Solutions to the Exercises, rather than just on its coefficients.)

Differentiating and integrating power series: Read 14.3 to p. 493 (middle).

Work: 7B-6acdeh; 7D-1ade, 2aefg, 3c

Recitation 15. Wed. Oct. 14 *(Tues. Oct. 13 is also a recitation day.)*

Introduction. to finite discrete random variables.

Read: Notes P, sections 1.1,1.2; 3 (through Example 3.1, for finite RV's only)

Work: 8A-1,2,3; 8C-1a

(Some solutions to Exercise Section 8 are right after Exercise section 8E.)

Part II (20 pts.)

Directions: Try each alone first; if you collaborate later, solutions must be written up independently. It is illegal to consult problem sets from previous years.

Problem 1. (Th. 6: 2,4)

a) 473/27a (What terms are missing? How does their sum relate to $\sum 1/n^2$?)

b) 469/27 (for each false statement, give a counterexample)

Problem 2. (Fri. 2 pts.) Find the power series representing $\ln\left(\frac{1-x}{1+x}\right)$ by combining the power series for $\ln(1-x)$ and $\ln(1+x)$.

Problem 3. (Fri., 4 pts.: 2,2) Work: 494/3a,d (see the book's examples: for (a), start by differentiating the series; for (d), start by integrating the series.)

Problem 4. (Fri. 2 pts.) Work 473/28

Problem 5. (Wed. 4 pts: 2,1,1)

In Hot Six, the Baby Einstein version of a Las Vegas slot machine, you feed in a play chip and press a button: three dials whirl around, each stopping at the end and showing a 1, 2, or 3.

If you add them up right and get a 6, you press the button, a light goes on and the machine gives you back four play chips. Anything else: no light, no chips, just a synthesized "sorry" and you can try again.

a) Let X be the finite discrete random variable whose values are the possible sums, from $x_3 = 3$ to $x_9 = 9$. Make up a two-row table showing x_i and $P(x_i)$, i=3,...,9, verify that $\sum_3^9 P(x_i) = 1$, and draw a histogram roughly to scale showing the seven probabilities.

b) If you play correctly all the time, will your pile of chips increase or decrease, and why? Find the mean $E(X)$ by inspection, and verify that $E(X) = \sum x_i P(x_i)$.

c) Using a hand calculator or computer (or even Google, which does arithmetic now and will find $\sqrt{x}$ for you), calculate the standard deviation $\sigma(X)$.

(As a check, we'll see that for bell-shaped histograms like this, usually about 2/3 of the area of the histogram lies over the interval of length $2\sigma(X)$ centered around the mean $E(X)$.)

Problem 6. (Wed. 2 pts.) You toss a fair coin up to four times: you get $\$n$, $n = 1, ..., 4$ if Heads (H) comes up for the first time on the nth toss; the game ends if you toss Tails (T) four times in a row.

How much should you pay each time to play this game if you expect to come out roughly even, money-wise, in the long run?