18.01A Practice Exam 1 Solutions

1 Using alg. methods:

$$\frac{e^{-x}}{1+2x} = \left(1-x+\frac{x^2}{2}\dots\right)\left(1-2x+4x^2\dots\right)$$
$$= 1-3x+x^2\left(\tfrac{1}{2}+2+4\right)+\dots$$

linear: $1-3x$

quad: $1-3x+\frac{13}{2}x^2$

2 a) $$\lim_{x\to 0}\frac{\sin^2 x}{1-\cos x} \underset{\text{L'Hosp}}{=} \lim_{x\to 0}\frac{2\sin x\cos x}{\sin x} = 2$$

b) $$\int_0^1 2x\cos(3x^2)\,dx = \frac{1}{3}\sin(3x^2)\Big]_0^1 = \frac{\sin 3}{3}$$

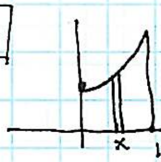
3 $$\int_0^{\sqrt{2}/2}\frac{dx}{(1-x^2)^{3/2}} = \int_0^{\pi/4}\frac{\cos u\,du}{\cos^3 u}$$

$x=\sin u$ $\quad dx = \cos u\,du$

$1-x^2=\cos^2 u$ $\quad \sin \pi/4 = \frac{\sqrt{2}}{2}$

$$= \int_0^{\pi/4}\sec^2 u\,du \quad \left(\text{or } \int_0^{\pi/4}\frac{du}{\cos^2 u}\right)$$

4



$$\text{vol.} = \int_0^1 2\pi x y\,dx = \int_0^1 2\pi x e^x\,dx$$

$$\int_0^1 xe^x\,dx \underset{\text{int by parts}}{=} xe^x\Big]_0^1 - \int_0^1 e^x\,dx = e - e^x\Big]_0^1 = 1$$

Ans: 2π

5 $$F(x) = \int_0^x \frac{4-t^2}{4+t^2}\,dt$$

a) $F(x)$ defined for all x, since integrand is continuous for all t: $(4+t^2>0)$

$$F'(x) = \frac{4-x^2}{4+x^2} \geq 0 \iff 4-x^2\geq 0 \iff \boxed{-2\leq x\leq 2}$$

$F'(x)=0$ when $\underset{\text{max}}{x=2}$ or $\underset{\text{min}}{x=-2}$

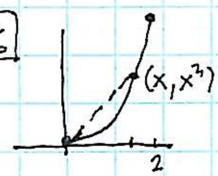
(max, min since $F(x)$ increasing on $[-2,2]$)

b) $$\int_0^x\frac{1-u^2}{1+u^2}\,du = \int_0^x\frac{4-4u^2}{4+4u^2}\,du$$

Set $t=2u$, $dt=2du$

$$= \int_0^{2x}\frac{4-t^2}{4+t^2}\,\frac{dt}{2} = \frac{F(2x)}{2}$$

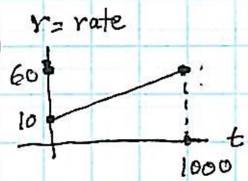
6



Average distance

$$= \frac{1}{2}\int_0^2\sqrt{x^2+x^4}\,dx = \frac{1}{2}\int_0^2 x\sqrt{1+x^2}\,dx$$
$$= \frac{1}{6}(1+x^2)^{3/2}\Big]_0^2 = \frac{1}{6}\left[5\sqrt{5}-1\right]$$

7 a)



$$r = \frac{50t}{1000}+10 = \frac{t}{20}+10$$

In $[t_i, t_i+\Delta t]$,

amt drug produced $\approx \left(\frac{t_i}{20}+10\right)\Delta t$ kg

refrigerate for $(1000-t_i)$ hours @ .01/kg/hr

cost $\approx \underbrace{\left(\frac{t_i}{20}+10\right)}_{\text{kg}}\underbrace{(1000-t_i)}_{\text{hours}}(.01)$

b) Adding & passing to limit as $\Delta t\to 0$:

$$\int_0^{1000}\left(\frac{t}{20}+10\right)(1000-t)(.01)\,dt$$