Directions: You can collaborate, but should list those you worked with and write up the solutions independently (i.e., not copying but thinking through by yourself).
Consulting solutions to problem sets of previous semesters is not allowed.

Reading: Mon. Chapter 3.1-5 Definition of limit, examples, infinite limits, \( \lim a^n \).

Problem 1. (2.5: .5, 1, 1) Do these three exercises (end of Chapter 3) using just the definition of limit in Chapter 3 (i.e., without using any limit theorems you may have learned in calculus, or just feel are intuitively obvious, like \( \lim 1/n = 0 \)).
   a) Work 3.1/2 (you can use Q2.1/3(i) – Ass’t 2, Problem 1a));
   b) Work 3.2/1 (use the triangle inequality and \( K – \epsilon \) principle)
   c) Work 3.3/3 (using Def’n 3.3 of limit \( \infty \))

Problem 2. (2 pts: 1 1))
   a) Prove that if \( \{x_n\} \) converges, it is bounded for \( n \gg 1 \).
   b) Then prove that it is bounded (i.e., for all \( n \))
      (For part (a), you’ll need an \( \epsilon \); tell who gets to choose it, and the reason for your answer. Also, the equivalent form of \( a_n \approx \epsilon L \) suggested by [2.4, (2)] is helpful.)

Problem 3. (1.5) a) Work 3.4/3 (at the end of the chapter).
   (Follow the hint given; note that the proof of Theorem 3.4 could be simplified slightly by dropping the 1 in addition to the other terms dropped.)

Problem 4. (2) Work P3-5(i) (p.48), just for the strictly increasing case, and without using infinite decimals.
   (Follow the hint given. Show that your construction of the sequence \( a_n \) guarantees that it will be strictly increasing, will have \( c \) as its limit (in the sense of Def’n 3.1), and the \( a_n \) will all be rational numbers.)

Reading: Wed. Chap. 3.6-.7, 4.1-2 Limits of integrals; the error form for limits; examples of its use; sequences derived from geometric progressions.

Problem 5. (2 pts.) Work 3.6/1b: prove \( \lim_{n \to \infty} \int_2^3 \ln^n x \, dx = \infty \). (Use the hints there).

Problem 6. (2) Let \( a > 1 \). Prove \( a^{1/n} \to 1 \) as \( n \to \infty \), by writing the sequence \( \{a^{1/n}\} \) in the error form and proving the error term \( e_n \to 0 \).
   (The ideas in the proof of Thm. 3.4 will be helpful.)

Problem 7. (3 pts.: 1, 2) Work 4.2/1ab.
This beautiful result was discovered by Leibniz at the age of 17. It led him to devote his early life to mathematics; he later switched to philosophy. He and Newton independently invented Calculus, but it is Leibniz’ notation we mostly use today.

The proof is a good illustration of estimating definite integrals rather than trying to calculate them exactly.