Solutions

1. a) $-2xe^{-x^2}\tan x + e^{-x^2}\sec^2 x$

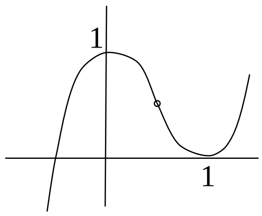
   b) $f'(u) = 2\cos(u^2+2u)\cdot -\sin(u^2+2u)\cdot(2u+2)$; $f'(1) = -2\cos 3\cdot\sin 3\cdot 4 = -4\sin 6$

   c) $\left.\dfrac{(x^2+1)\cdot 2(2x+1)\cdot 2-(2x+1)^2\cdot 2x}{(x^2+1)^2}\right|_{x=1} = \dfrac{24-18}{4} = \dfrac{3}{2}$

2. Differentiating implicitly, $y^3+3xy^2y'-4xy-2x^2y'+5=0$

   $\Rightarrow\quad 1+6y'-8-8y'+5=0$ at $(2,1)$ $\Rightarrow$ $y'(2)=-1$;

   tangent line: $y-1=-(x-2)$, or $y=-x+3$.



3. $y' = 6x^2-6x$; $=0$ if $x=0,1$; $y(0)=1$, max; $y(1)=0$, min

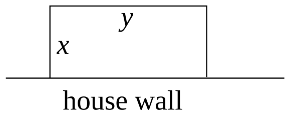
   $y''=12x-6$; $=0$ if $x=\frac{1}{2}$; $y(\frac{1}{2})=\frac{1}{2}$, point of inflection

4. Maximize $A=xy$, where $2x+y=48$.

   $A=xy=x(48-2x)=48x-2x^2 \quad\Rightarrow\quad A'=48-4x$;

   $A'=0$ if $x=12$; then $y=24$, and $A=288$ sq. feet.

   Since $x=0$ or $y=0$ gives $A=0$, the above must be a maximum.



5. $\displaystyle\lim_{h\to 0}\frac{(10+h)^7-10^7}{h} = \left.\frac{d}{dx}x^7\right|_{x=10} = 7\cdot 10^6$

6. a) cont., but not diff. b) neither, since undefined at $x=0$
   c) both, since the two functions have the same value 1 and the same derivative 0 when $x=0$.

7. Let $V=$ volume of oil, $r=$ disc radius, $h=$ disc thickness; then $V=\pi r^2 h$.

   Using mks units: data: when $r=1$, $\dfrac{dh}{dt}=-.004$; to find: $\dfrac{dh}{dt}$, when $r=2$.

   $h=\dfrac{V}{\pi r^2} \;\Rightarrow\; \dfrac{dh}{dt} = -\dfrac{2V}{\pi r^3}\cdot\dfrac{dr}{dt}$. Substituting in the data: $-.004 = -\dfrac{2V}{\pi}\cdot .1 \;\Rightarrow\; \dfrac{2V}{\pi} = .04$.

   Therefore when $r=2$, $\dfrac{dh}{dt} = -\dfrac{2V}{\pi\cdot 8}\cdot .1 = -\ .0005$; so thickness decreases at .5 mm/sec.

8. Separating variables, the equation becomes $e^y\,dy = e^{2x}\,dx$.

   Integrating both sides, $e^y=\frac{1}{2}e^{2x}+c$; substituting, $y(0)=0 \Rightarrow c=\frac{1}{2}$.

   Solving for y, $y=\ln\frac{1}{2}(e^{2x}+1) = \ln(e^{2x}+1)-\ln 2$.

9. $\displaystyle\int_0^1 x^3(x^4+3)^{1/2}\,dx = \frac{1}{6}(x^4+3)^{3/2}\Big]_0^1 = \frac{1}{6}(8-3\sqrt{3})$. (Put $u=x^4+3$, so $du=4x^3$.)

10. Graph is above x-axis for $x\ge 1$. Area $= \displaystyle\int_1^2\frac{\ln^3 x}{x}\,dx = \frac{1}{4}\ln^4 x\Big]_1^2 = \frac{1}{4}(\ln 2)^4$.

11. Volume $= \displaystyle\int_0^a \pi\cdot\frac{1}{(2x+1)^2}\,dx = \frac{-\pi/2}{2x+1}\Big]_0^a = \frac{\pi}{2}\left(1-\frac{1}{2a+1}\right)$, which $\to \dfrac{\pi}{2}$, as $a\to\infty$.

12. $\displaystyle\int_0^{\pi/2}\sin x\,dx \approx \frac{\pi}{4}\left(\frac{1}{2}\sin 0+\sin\frac{\pi}{4}+\frac{1}{2}\sin\frac{\pi}{2}\right) = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right) \approx 3.1\cdot\frac{2.4}{8}\approx .9$