PROBLEM SET 3 (DUE ON OCT 6)

(All Exercises are references to the December 29, 2015 version of *Foundations of Algebraic Geometry* by R. Vakil.)

**Problem 1.** Exercise 3.7.E (irreducible closed subsets correspond to prime ideals)

**Problem 2.** Prove that if Spec $A$ is disconnected then $A$ is isomorphic to the product of nonzero rings $A_1$ and $A_2$. (Hint: use the sheaf axioms for the structure sheaf of Spec $A$. You might also find Remark 3.6.3 helpful.)

**Problem 3.** Let $X = \text{Spec } k[x, y, z]/(xz, yz)$ and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$. Is $U$ a distinguished open set?

**Problem 4.** Exercise 4.3.A (classifying isomorphisms of affine schemes)

**Problem 5.** Exercise 4.3.G (functions on locally ringed spaces)

**Problem 6.** Let $X_1 = \text{Spec } k[x, y]$ and $X_2 = \text{Spec } k[w, z]$ be two copies of the affine plane over a field $k$. Let $X$ be the scheme formed by gluing $X_1$ and $X_2$ along the isomorphism of open subschemes $\text{Spec } k[x, x^{-1}, y] \cong \text{Spec } k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of $X$. Is $X$ affine?