PROBLEM SET 2 (DUE ON SEP 29)

(All Exercises are references to the December 29, 2015 version of Foundations of Algebraic Geometry by R. Vakil.)

Problem 1. Exercise 3.2.Q (picturing $\mathbb{A}^n_\mathbb{Z}$)
Problem 2. Describe the map of spectra induced by the ring homomorphism $\mathbb{Q}[x] \to \mathbb{C}[x]$ sending $x$ to $x$. What is the preimage of the generic point?
Problem 3. Describe $\text{Spec } \mathbb{C}[x, y, z]/(xy, yz, zx)$ (as a topological space).
Problem 4. Let $A$ be a ring and let $S$ be a multiplicative subset of $A$. Consider the localization map $\phi : A \to S^{-1}A$ and the induced map on spectra $\text{Spec } \phi : \text{Spec } S^{-1}A \to \text{Spec } A$. Show that $\text{Spec } \phi$ is injective and that the Zariski topology on $\text{Spec } S^{-1}A$ agrees with the subspace topology induced by this injection.
Problem 5. Exercise 3.5.B (covering $\text{Spec } A$ with distinguished open sets)
Problem 6. Exercise 3.5.E (equivalent conditions to $D(f) \subset D(g)$)
Problem 7. Exercise 3.6.J (when are the closed points dense?)
Problem 8. Exercise 3.6.K (sometimes functions are determined by their values on closed points)