PROBLEM SET 7 (DUE ON NOV 3)

(All Exercises are references to the December 29, 2015 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 8.1.H (closed subschemes correspond to quasicoherent ideal sheaves)

Problem 2. A *quadric* in $\mathbb{P}^n_k$ is a closed subscheme cut out by a single homogeneous polynomial of degree two (see 8.2.2 in the notes). Give an example of two quadrics in $\mathbb{P}^2_{\mathbb{R}}$ intersecting in a single point, and compute the scheme-theoretic intersection. Then give a second example of this, with scheme-theoretic intersection not isomorphic (as schemes) to that in your first example.

Problem 3. Exercise 8.2.C (closed subschemes of projective schemes are projective)

Problem 4. Exercise 8.3.A (scheme-theoretic image of a reduced scheme is reduced)

Problem 5. Let $\pi : \mathbb{P}^2_{\mathbb{C}} \to \mathbb{P}^3_{\mathbb{C}}$ be induced by the graded ring homomorphism $\mathbb{C}[y_0, y_1, y_2, y_3] \to \mathbb{C}[x_0, x_1, x_2]$ sending y_0, y_1, y_2, y_3 to $x_0^2, x_1^2, x_2^2, x_0x_1 + x_0x_2$. Show that π factors through the Veronese embedding $\mathbb{P}^2_{\mathbb{C}} \to \mathbb{P}^5_{\mathbb{C}}$ in the sense that there exists an open set $U \subset \mathbb{P}^5_{\mathbb{C}}$ containing the image of the Veronese embedding and a morphism $U \to \mathbb{P}^3_{\mathbb{C}}$ that composes with the Veronese embedding of $\mathbb{P}^2_{\mathbb{C}}$ in U to give π. Also show that the scheme-theoretic image of π is a hypersurface of degree 4 in $\mathbb{P}^3_{\mathbb{C}}$.