PROBLEM SET 2 (DUE ON SEP 29)

(All Exercises are references to the December 29, 2015 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 3.2.Q (picturing $\mathbb{A}^n_{\mathbb{Z}}$)

Problem 2. Describe the map of spectra induced by the ring homomorphism $\mathbb{Q}[x] \to \mathbb{C}[x]$ sending x to x. What is the preimage of the generic point?

Problem 3. Describe $\operatorname{Spec} \mathbb{C}[x, y, z]/(xy, yz, zx)$ (as a topological space).

Problem 4. Let A be a ring and let S be a multiplicative subset of A. Consider the localization map $\phi : A \to S^{-1}A$ and the induced map on spectra $\operatorname{Spec} \phi : \operatorname{Spec} S^{-1}A \to \operatorname{Spec} A$. Show that $\operatorname{Spec} \phi$ is injective and that the Zariski topology on $\operatorname{Spec} S^{-1}A$ agrees with the subspace topology induced by this injection.

Problem 5. Exercise 3.5.B (covering $\operatorname{Spec} A$ with distinguished open sets)

Problem 6. Exercise 3.5.E (equivalent conditions to $D(f) \subset D(g)$)

Problem 7. Exercise 3.6.J (when are the closed points dense?)

Problem 8. Exercise 3.6.K (sometimes functions are determined by their values on closed points)