Tools of gauge theory have been used since the 1980s to produce exotic 4-manifolds—smooth manifolds that are homeomorphic but not diffeomorphic. In principle, the same circle of ideas can be used to study the diffeomorphism group of a 4-manifold, but concrete examples and calculations are hard to come by. I will give examples of elements of infinite order in the fundamental group of the diffeomorphism group of some basic simply connected 4-manifolds. This is joint work with Dave Auckly.