We develop an approach to the study of the configuration spaces of a cell complex $X$ that is both flexible and suitable for computation. We proceed by viewing $X$, together with its subdivisions, as a “subdivisional space,” a kind of diagram object, which has associated to it certain diagrammatic versions of configuration spaces. These objects, which model the correct homotopy types, mix the discrete and the continuous, and they may be attacked by combining techniques drawn from discrete Morse theory and factorization homology. We apply our theory in the 1-dimensional example of a graph, obtaining an enhanced version of a family of chain models for graph braid groups originally studied by Swiatkowski. These complexes come equipped with a robust computational toolkit, which we exploit in numerous calculations, old and new. This is joint work with Byung Hee An and Gabriel Drummond-Cole.