In 2002, Malle formulated a conjecture regarding the distribution of number fields with specified Galois group. The conjecture is an enormous strengthening of the inverse Galois problem; it is known to hold for abelian Galois groups, but for very few non-abelian groups.

We may reformulate Malle’s conjecture in the function field setting, where it becomes a question about the number of branched covers of the affine line (over a finite field) with specified Galois group. In joint work with Jordan Ellenberg and TriThang Tran, we have shown that the upper bound in Malle's conjecture does hold in this setting.

The techniques used involve a computation of the cohomology of the (complex points of the) Hurwitz moduli spaces of these branched covers. Surprisingly (at least to me), these cohomology computations can be rephrased in terms of the homological algebra of certain braided Hopf algebras arising in combinatorial representation theory and the classification of Hopf algebras. This relationship can be leveraged to provide the upper bound in Malle’s conjecture.